

EXPERIMENTAL INVESTIGATION AND ANALYSIS OF FRICTION TORQUE IN TAPERED ROLLER BEARINGS

A Thesis submitted to Gujarat Technological University

for the Award of

Doctor of Philosophy

in

Mechanical Engineering

by

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under supervision of

Dr. Jaydeep K. Dadhaniya



**GUJARAT TECHNOLOGICAL UNIVERSITY
AHMEDABAD**

October 2026

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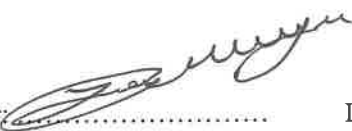
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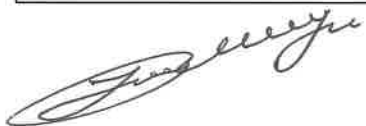
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ABSTRACT

Tapered roller bearings (TRBs) are widely employed in automotive, railway, heavy machinery, and industrial transmission systems due to their capability of carrying both radial and axial loads. However, under high axial loading conditions, the friction torque generated at the roller raceway and roller end rib contacts increase considerably, resulting in excessive heat generation, elevated bearing temperature, increased power loss, lubricant degradation, and reduced bearing service life. Although numerous studies have investigated the influence of operating conditions on bearing performance, the effect of inner race rib convexity on the frictional and thermal behaviour of TRBs has received limited attention. Therefore, the present research aims to experimentally investigate the influence of inner race rib convexity, axial load, and operating speed on friction torque and bearing temperature, and to determine their optimum combination for improving bearing performance.

To accomplish the research objectives, a dedicated friction torque test rig was used for conducting controlled experiments on single row TRBs 33118. The test rig consists of a drive unit, customized bearing hub, hydraulic loading system, flexible coupling arrangement, torque sensor, and a SCADA based data acquisition system capable of continuously monitoring friction torque, bearing temperature, axial load and rotational speed. The performance of the developed test rig was validated by comparing the experimentally obtained bearing temperature characteristics with the standard SKF bearing temperature profile, and a close agreement between the two confirmed the accuracy and reliability of the experimental setup.

Prior to the main experimentation, pilot experiments were conducted over a wide range of axial loads, operating speeds, and rib convexity values to establish the safe operating limits of the bearing and test rig. The pilot study indicated that operation beyond 6000 kgf axial load and 600 rpm resulted in excessive friction torque, rapid temperature rise, severe vibration, and noticeable surface damage on the bearing components. Post-test inspection confirmed that operation up to 6000 kgf maintained satisfactory bearing condition with only slight thermal discoloration. In addition, Hertzian contact stress analysis using MESYS AG software verified that rib convexity values up to 6 μm maintained the maximum contact stress below approximately 2400 MPa, thereby ensuring safe operating conditions. Based on these investigations, three process parameters, namely rib convexity (0, 3, and 6 μm), axial load (2000, 4000, and 6000 kgf), and

operating speed (400, 500, and 600 rpm), were selected for the detailed experimental study.

Initially, a three factors, three levels, L27 Full Factorial Design (FFD) was employed to investigate the combined influence of the selected parameters on friction torque and bearing temperature. The experimental data were analysed using MINITAB 19 to develop second-order regression models and perform Analysis of Variance (ANOVA). The developed regression models exhibited excellent predictive capability with coefficients of determination (R^2) of 98.87% for friction torque and 99.36% for bearing temperature. Statistical analysis confirmed that rib convexity, axial load, and operating speed significantly influence both responses. The experimental results revealed that friction torque increases with increasing axial load and operating speed, whereas it decreases with increasing rib convexity. However, bearing temperature attained its minimum value at approximately 3 μm rib convexity, indicating an optimum rib geometry that balances friction reduction and contact stress distribution.

To improve modelling efficiency and reduce the number of experimental runs, the Design of Response Surface Methodology based Box Behnken (RSM BBD) was subsequently adopted. The developed RSM BBD models showed high predictive capability, with R^2 values of 99.74% for friction torque and 99.21% for bearing temperature. Contour plots and three dimensional response surface plots were used to investigate the interaction effects among rib convexity, axial load, and operating speed, providing a clear understanding of their combined influence on friction torque and bearing temperature. A comparative assessment between the Full Factorial Design and the RSM BBD indicated that both techniques generated statistically reliable models however, the Box–Behnken Design required 15 experimental runs as compared to 27 runs of Full Factorial Design, while maintaining comparable modelling accuracy, making it more suitable for optimization studies involving tapered roller bearings.

Multi response optimization was carried out using four different approaches, namely Full Factorial optimization using MINITAB 19, Desirability Function Approach (DFA), RSM based BBD, and Grey Relational Analysis (GRA). The optimum operating conditions predicted by all four optimization techniques were found to be in close agreement. The RSM based optimization yielded the highest desirability and predicted the optimum combination at approximately 3.21 μm rib convexity, 2000 kgf axial load and 400 rpm, yielding a predicted minimum friction torque of 7.96 Nm and a bearing temperature of 54.57 °C. Confirmation experiments conducted at the optimum condition demonstrated good agreement with the predicted results, thereby validating

the developed regression models and optimization methodologies.

Overall, A novel experimental and predictive framework integrating Full Factorial Design, Response Surface Methodology based Box–Behnken Design (RSM-BBD), Desirability Function Analysis (DFA), and Grey Relational Analysis (GRA) was developed for multi response optimization of TRB performance. The developed RSM-BBD models demonstrated predictive capability, with R^2 values of 99.74% for friction torque and 99.21% for bearing temperature. The optimized condition was identified as a rib convexity of 3.21 μm , axial load of 2000 kgf (19.61 kN), and rotational speed of 400 rpm, resulting in predicted friction torque of 7.96 Nm and bearing temperature of 54.57 °C with a composite desirability of 1.000. Confirmation experiments showed prediction deviations within 5%, demonstrating the applicability of the developed models and optimization methodology. The findings of this research offer practical guidelines for minimizing frictional losses, reducing operating temperature results in improving lubrication performance, enhancing the efficiency, reliability, and service life of TRBs used in automotive and industrial applications.

Keywords: Tapered Roller Bearing, Inner Race Rib Convexity, Friction Torque, Bearing Temperature, Experimental Investigation, Full Factorial Design, Response Surface Methodology, Box–Behnken Design, Regression Analysis, ANOVA, Desirability Function Approach, Grey Relational Analysis, Multi-response Optimization.

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List of Abbreviation

Abbreviations	Full Form
FT	Friction Torque
BT	Bearing Temperature
DOE	Design of Experiments
RSM	Response Surface Methodology
BBD	Box–Behnken Design
FFD	Full Factorial Design
DFA	Desirability Function Analysis
GRA	Grey Relational Analysis
TRB	Tapered roller bearings
GRG	Grey Relational Grade
GRC	Grey Relational Coefficient
MRO	Multi-Response Optimization
SCADA	Supervisory Control and Data Acquisition
SOP	Standard Operating Procedure
rpm	Revolutions per Minute
ANOVA	Analysis of Variance

List of Symbols

Symbol	Description
A	Rib Convexity (coded factor)
B	Axial Load (coded factor)
C	Operating Speed (coded factor)
β_0	Regression Constant
β_i	Linear Regression Coefficient
β_{ij}	Interaction Regression Coefficient
d_i	Individual Desirability Index
D	Overall Desirability
L	Lower Limit
T	Target Value
U	Upper Limit
y	Response Variable
x_i	Independent Variable
α	Level of Significance
μm	Micrometre
$^{\circ}\text{C}$	Degree Celsius

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CHAPTER 1

Introduction

1.1 Background of the Work

Rolling Contact Bearings (RCBs) are essential machine components widely used in automotive, aerospace, manufacturing, and industrial machinery applications. Their primary function is to support rotating shafts while minimizing friction, supporting load and ensuring smooth power transmission (Harris & Creceius, 1986; Harris & Kotzalas, 2006b, 2006a) (Xiong et al., 2026). Among the various types of RCBs, TRBs are preferred because of their capability to withstand both radial and axial loads simultaneously. This unique characteristic makes them particularly suitable for automotive wheel hubs, gearboxes, transmissions, and heavy-duty industrial equipment where high load carrying capacity and operational reliability are required (Bercea et al., 1996; Chen et al., 2023; Ishaq Khan et al., 2025; Lee & An, 2024a).

The performance and service life of TRBs are strongly influenced by the friction generated at the rolling contacts between rollers and raceways. During operation, friction torque (FT) is developed due to the interaction of bearing components, lubricant behaviour, applied load, and operating conditions. Under high bearing loads and unfavourable operating conditions, excessive FT can result in significant heat buildup within the bearing system. The generated heat increases the bearing temperature (BT), accelerates lubricant degradation, reduces mechanical efficiency, and finally lead to premature bearing failure. Consequently, controlling FT and BT has become an important research area for improving bearing reliability and overall performance.

Previous studies have reported that bearing performance is significantly affected by geometric and operating parameters such as surface roughness, bearing load, rotational speed, lubrication conditions, and material properties. Variations in these parameters alter the contact characteristics between rolling elements and raceways, thereby influencing frictional torque generation and heat dissipation. Although considerable research has been

conducted on bearing tribology and thermal behaviour, identifying the optimal combination of operating parameters for minimizing FT and BT remains a challenging task.

To address this issue, the present work focuses on investigating the influence of selected operating parameters on the performance characteristics of TRBs. The study evaluates the effects of excessive axial loading, bearing speed and inner raceway rib convexity on FT and BT through a systematic experimental approach. Statistical and optimization techniques, including FFD, BBD RSM, Desirability Function Approach (DFA) and Grey Relational Analysis (GRA), are employed to analyse the responses and determine optimal operating conditions. The outcomes of this present research is expected to provide valuable contribution toward reducing frictional losses, controlling BT, improving bearing reliability, and enhancing the operational efficiency of automotive and industrial bearing applications.

1.2 Rolling Contact Bearings (RCBs)

The invention of the wheel marked a turning point in mechanical progress, revealing that rolling motion requires far less effort than sliding. Even after the development of lubrication to reduce sliding friction, rolling movement continued to demonstrate superior efficiency. Archaeological findings suggest that as early as 2400 BC, the Egyptians used lubrication—likely water—to ease the movement of heavy stone structures on sledges. Around 1100 BC, the Assyrians improved upon this principle by placing rollers beneath their sledges as shown in FIGURE 1.1, further reducing the manpower required for transport. These early practices laid the foundation for the development of rolling motion and, eventually, rolling bearings (RBs)(Harris & Kotzalas, 2006b).

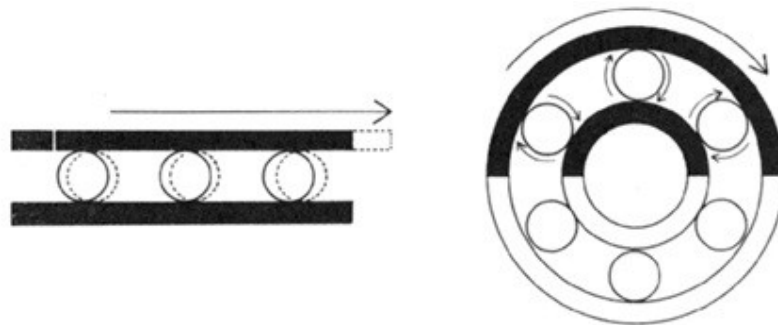


FIGURE 1.1 Progressions of RCBs (Harris & Kotzalas, 2006b)

Although the concept of rolling motion was recognized for millennia, its application in engineered mechanisms was limited by manufacturing constraints. A basic forms of RBs were known to exist around 50 AD during the Roman era, but the idea gained true engineering significance only with the work by Leonardo da Vinci (Reti, 1971). In his *Codex Madrid*, Leonardo sketched early designs of pivot and ball bearings featuring rolling elements and cages to maintain spacing—precursors of the modern bearing. Despite these insights, RCBs did not achieve widespread industrial use until the Industrial Revolution, when the demand for efficient mechanical systems intensified.

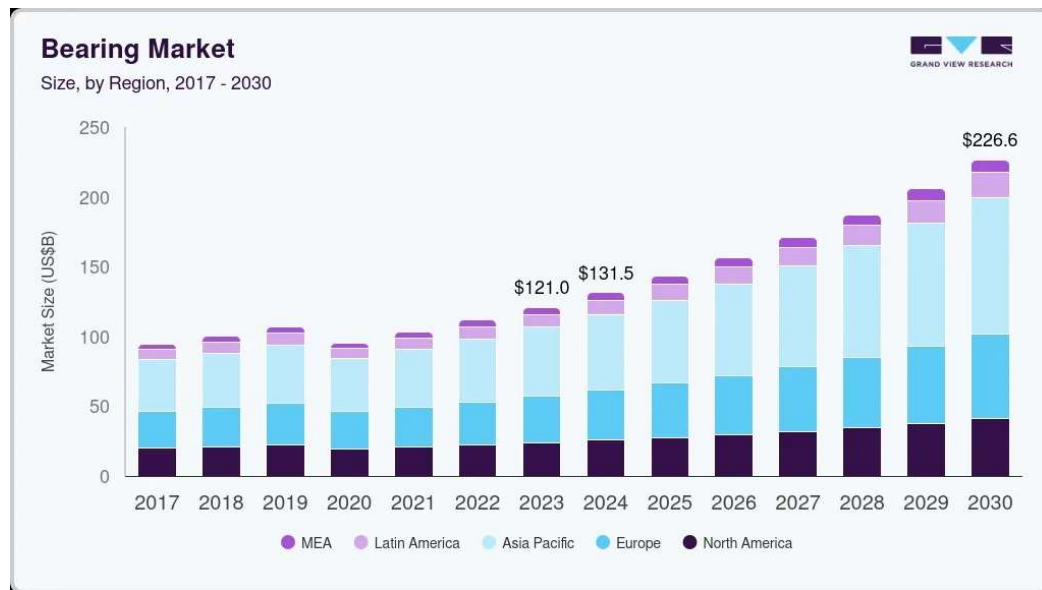


FIGURE 1.2 Roller Bearing Market Trend (*Bearing Market Size, Share & Growth Analysis Report, 2030, n.d.*)

The global bearing market trend as shown in FIGURE 1.2 showed strong and sustained growth, driven by increasing demand for efficient, durable, and low-maintenance mechanical components across multiple industries. With the market valued of USD 120.98 billions in 2023 and projected to reach USD 226.60 billions by the year 2030 (*Bearing Market Size, Share & Growth Analysis Report, 2030, n.d.*), advances in metallurgy, precision machining, and lubrication technology transformed RBs from simple devices into high-performance components. The development of high-quality bearing steels, accurate geometric tolerances, and sophisticated manufacturing processes especially after 1960 greatly enhanced their load carrying capacity, fatigue life, and reliability (Reti, 1964). These improvements solidified the RBs place as one of the most critical elements in modern rotating machinery.

RCBs are among the most widely used components in rotating machinery (Harris & Kotzalas, 2006b). Their main function is to support loads and minimize friction among moving parts by introducing rolling elements between two surfaces (SKF Groups, 2018). This configuration replaces sliding friction with rolling contact, significantly reducing energy losses, heat generation, and surface wear, while improving operational smoothness and efficiency. Their straightforward design, ease of installation and removal, and availability in standardized sizes contribute to their extensive industrial adoption.

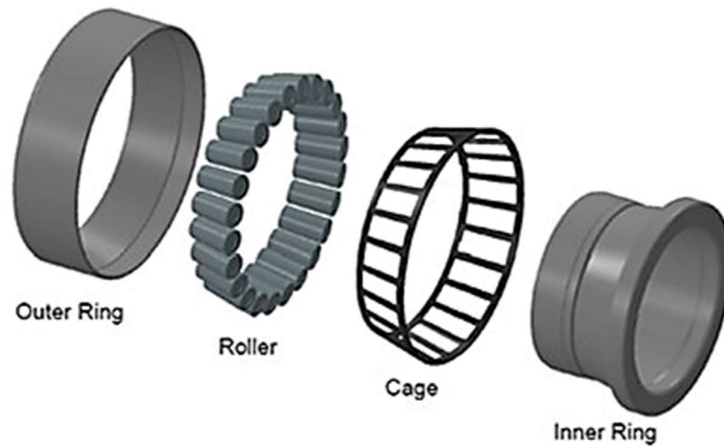


FIGURE 1.3 Components of Rolling Contact Bearing

A typical RBs consists of four essential components as an inner ring, an outer ring, rolling elements and a cage (Bisson & Anderson, 1964) as shown in FIGURE 1.3. The inner as well as outer rings provide the raceways for the rolling elements, which may be spherical or cylindrical depending on the design. The cage maintains uniform spacing between the rotating elements, ensuring smooth operation and preventing direct contact. Typically, the inner ring rotates together with the shaft, while the outer ring is held stationary within a housing; however, in certain configurations, both rings may rotate relative to one another. Lubrication—using either grease or oil—is essential to minimize friction, dissipate heat, and prolong bearing life.

RBs are broadly categorized into two main types: ball bearings and roller bearings as shown in FIGURE 1.4. Ball bearings exhibit low rolling resistance and are therefore ideal for high-speed applications, though they are less suited for heavy loads. Roller bearings, in contrast, can accommodate higher radial or axial loads due to a larger contact area, but they generally operate at lower speeds. Bearings are also classified according to the direction of the applied

load: radial bearings are designed for forces acting perpendicular to the shaft, while thrust bearings handle axial loads. Some specialized configurations can accommodate combined radial and axial forces (Harris & Kotzalas, 2006a).

Among ball bearings, the single row deep groove ball bearings are the most commonly used type. They are capable of carrying both radial and limited axial loads while maintaining excellent performance at high speeds. Double row ball bearings provide greater load capacity, while angular contact ball bearings (ACBB) are intended to carry significant thrust loads in one or both directions. Self-aligning ball bearings, featuring a spherical outer raceway, allow for slight misalignment between the shaft and housing and are commonly used in flexible systems. Thrust ball bearings, conversely, are specialized for high speed axial load applications.

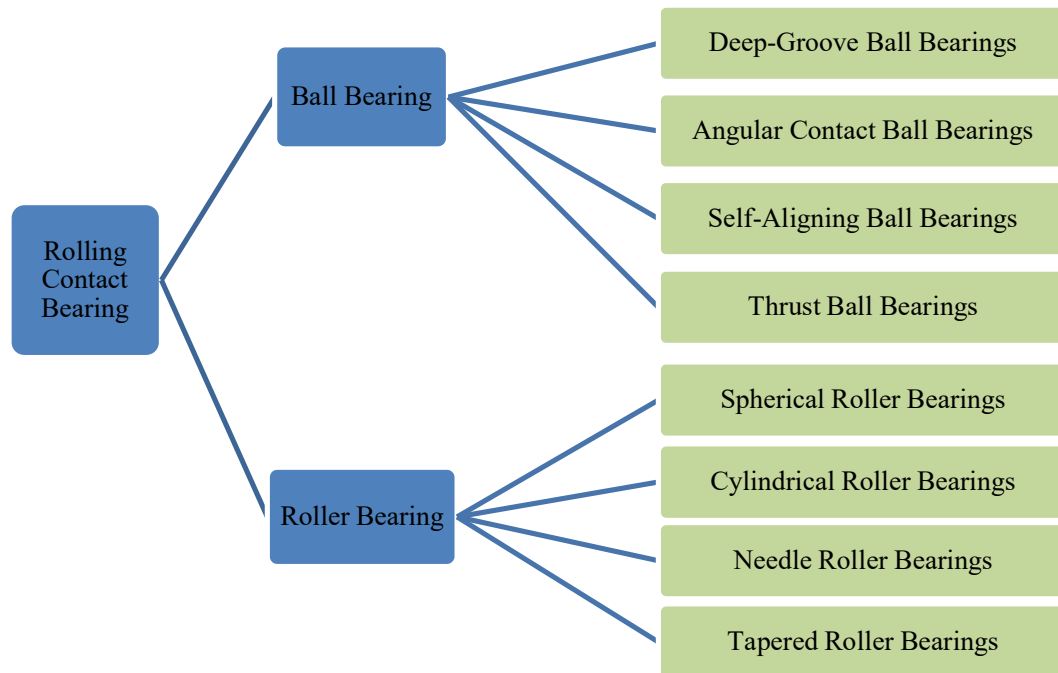


FIGURE 1.4 Classification of Rolling Contact Bearings (RCBs)

In the roller bearing category, cylindrical roller bearings are the most prevalent, capable of supporting heavy radial loads and functioning effectively at high speeds. Needle roller bearings, characterized by their slender rolling elements, are ideal for compact assemblies requiring high load capacity. Tapered roller bearings (TRBs) are designed to handle combined radial and axial loads but are typically operated in pairs to balance opposing thrust forces. Spherical roller bearings, with their self-aligning capability, perform well under high

loads even in the presence of misalignment. Thrust roller bearings—which may contain cylindrical, spherical, or tapered rollers—are employed for high axial loads, although they are not recommended for very high-speed applications. The dimensions and tolerances of RBs are standardized by the International Organization for Standardization (ISO) to ensure interchangeability among manufacturers. Standard parameters include bore size, outer diameter, bearing width, and chamfer dimensions. This global standardization allows designers and engineers to replace or upgrade bearings without compatibility issues, simplifying maintenance and enhancing reliability (Cornish et al., 1973).

1.3 Introduction to Tapered Roller Bearings (TRBs)

TRBs are a specialized class of RCBs designed to accommodate both radial and axial loads simultaneously. Their distinguishing feature is the use of conical rollers and raceways, which are arranged such that the extended axes of all rolling elements intersect at a common point on the bearing axis. This geometric alignment ensures pure rolling motion, minimizes sliding friction, and enhances load-carrying capability. TRBs are widely employed in applications where heavy, shock, or combined loads occur, such as automotive wheel hubs, gearboxes, railway axles, construction equipment, industrial rotating machinery and transmission systems (Bisson & Anderson, 1964; Cornish et al., 1973; Harris & Kotzalas, 2006b, 2006a) (W. Zhang & Li, 2025a) . The inherent robustness, high stiffness, and ability to maintain accurate shaft alignment make TRBs preferable in systems demanding high reliability and long service life. Additionally, their adjustable internal clearance enables precise control of operating conditions, allowing engineers to optimize bearing performance for specific mechanical requirements.

The increasing adoption of electric commercial vehicles, expansion of industrial machinery production, and rising demand for energy-efficient components are among the key drivers influencing TRB development and deployment. A notable trend within the industry is the push toward low-friction and high-performance bearings, aligning with global emphasis on energy efficiency, reduced carbon emissions, and enhanced mechanical reliability.

From a market perspective, the global TRB industry has demonstrated as FIGURE 1.5 consistent expansion over the last several years. The market size reached USD 11.04 billion in 2024, with projections indicating growth to USD 11.19 billion in 2025 at a CAGR (Compounded Annual Growth Rate) of 1.4%. Over the longer forecast horizon, the market

is expected to reach USD 12.33 billion by 2029 with a CAGR of 2.4% (Grand View Research Inc., 2025).

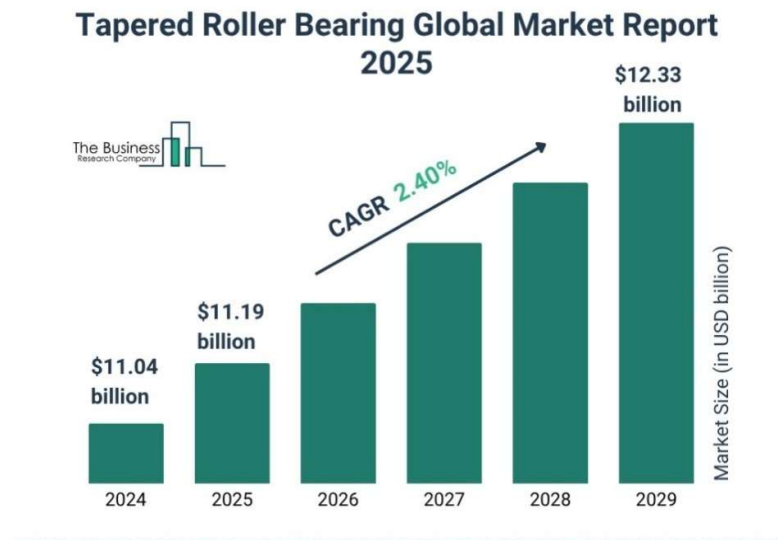


FIGURE 1.5 TRBs Global Market Trend (Grand View Research Inc., 2025)

However, despite these unique features and the increasing demand for TRBs in automotive and industrial applications, frictional torque remains an important concern during operation. Under high bearing loads, increased frictional torque generates excessive heat, which raises the bearing temperature and can lead to reduced efficiency, degraded performance, and premature bearing failure. The magnitude of frictional torque is influenced by several geometric and operating parameters, making it essential to understand and control their effects. Therefore, investigating the relationship between these parameters and frictional torque is important for improving bearing performance and reliability and for establishing optimized operating conditions for TRBs used in automotive and industrial applications.

1.4 Components of TRBs

TRBs are commonly manufactured in single-row, double-row, and four-row configurations and are available in a wide range of dimensional categories, including < 200 mm, 200 to 300 mm, 300 to 600 mm, and > 600 mm, depending on operational requirements. A TRB consists of mainly four key components, each playing a critical role in the bearing's function, load distribution, and overall performance. FIGURE 1.6 illustrates the principal components of a TRB that are the cup (outer race), cone (inner race), tapered shaped rolling elements and

bearing cage. The bearing is lubricated using either oil or grease to ensure smooth operation and minimize friction. The rolling elements retained in the bearing cage that locates between the cone and cup. During assembly, the cone is mounted on the rotating shaft, whereas the cup is securely fitted within the bearing housing. Depending on the application and loading requirements, a pair of TRBs can be arranged in either an O or X configuration. (Nguyen-Schäfer, 2019).

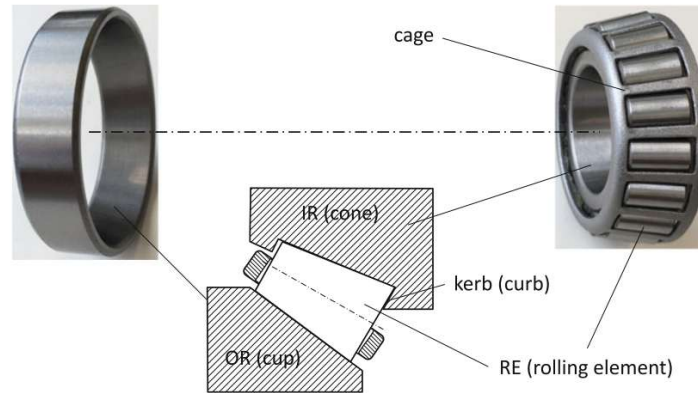


FIGURE 1.6 Components of a Single Row TRB (Nguyen-Schäfer, 2019)

1.4.1 Outer Ring (Cup)

The cup that is also known as the outer raceway of the bearing. It contains a tapered internal surface against which the rollers rotate. The angle and precision of the cup raceway determine the bearing's axial load capacity and influence rolling behaviour.

1.4.2 Inner Ring (Cone)

The cone is the inner raceway and incorporates a tapered outer surface matching the angle of the cup. It provides the seating surface for the rollers and includes a cone back face, which serves as a reference surface for setting axial preload or endplay during installation.

1.4.3 Tapered Rollers

The rolling elements are frustum-shaped rollers designed to maintain line contact with both raceways. This geometry allows the bearing to support high radial and axial forces while reducing stress concentrations. The rollers are typically made from high-carbon chromium steel to offer maximum hardness, wear resistance, and fatigue strength.

1.4.4 Cage (Retainer)

The cage maintains uniform spacing between adjacent rollers, ensuring proper roller guidance and preventing direct roller-to-roller contact. It assists in maintaining roller alignment and enhances the stability of the bearing under dynamic loads. Common cage materials include pressed steel, brass, and polymer composites.

1.5 Geometry of TRBs

FIGURE 1.7 presents the principal geometries of the TRBs in solid cut section view while FIGURE 1.8 displays the detailed geometric configuration of the TRBs. FIGURE 1.8 shows that d represents the cone bore diameter, D denotes the cup outer diameter and T corresponds bearing width. D_m is the mean diameter of rolling element at the roller centre O_{Re} and L_{Re} is the roller length. D_{pw} denotes the pitch diameter measured at the centre O_{Re} . α_i corresponds half cone angle formed by the cone and the axis of rotation x . Similarly, α_o is the half cup angle for OR. α_m denotes the half roller centre line angle and α_{12} is the half roller angle for the rotating elements.

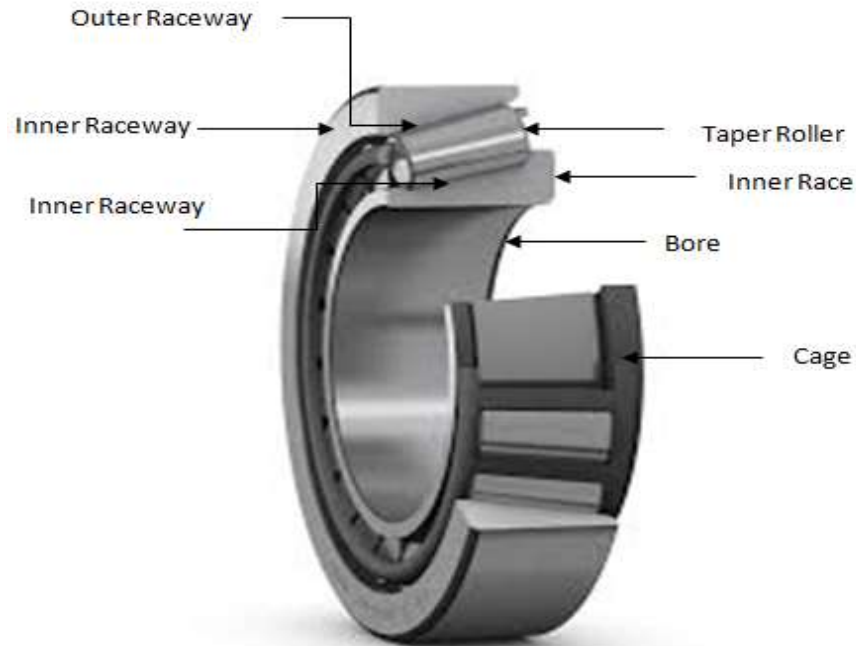


FIGURE 1.7 TRB Solid Cut Section (30207 - *Tapered Roller Bearings* | SKF, n.d.)

Finally, α_f is defined as kerb (or curb) angle between the rotating axis x and bearing kerb. Each row of the tapered roller bearing is composed of Z rolling elements. Depending on the axial (thrust) load to be supported, the bearing may employ a small or a steep contact angle,

as presented in FIGURE 1.8 The cone rib acts as a thrust face that guides the rollers and prevents them from shifting axially. It also transmits axial forces through the rollers into the raceways. Rib geometry and surface finish significantly influence frictional torque and heat generation.

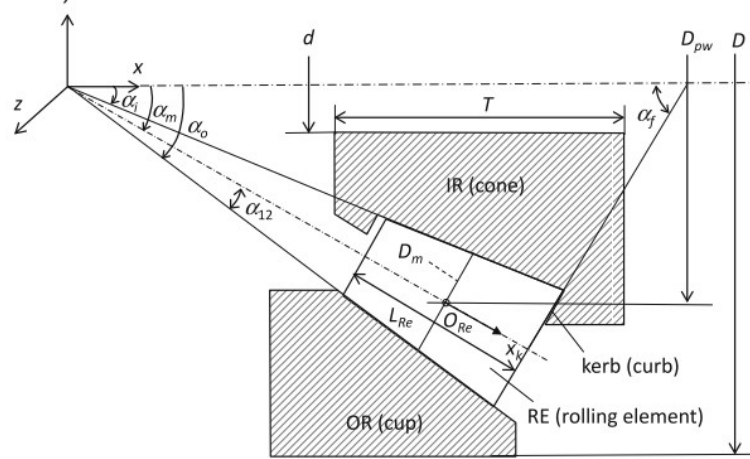


FIGURE 1.8 Detailed Geometry of TRBs (Harris & Kotzalas, 2006b)

1.5.1 Roller & Raceway Contacts

The roller–raceway contact geometry as shown in FIGURE 1.9 defines how loads and friction develop between the roller and the bearing races. As described in (W. Zhang & Li, 2025a), the roller interacts with the inner and outer raceways through oblique contact lines governed by roller crowning, curvature radii, and small tilts or offsets, which establish the Hertzian contact zones and resulting force–moment balance. These geometric features influence stress distribution, stiffness, and friction (Zhou & Hoeprich, 1991). According to (Matsuyama & Kamamoto, 2001), friction in this contact mainly comes from viscous rolling resistance, which is much larger than elastic hysteresis loss. Thus, the contact geometry, by shaping deformation and lubrication conditions, plays a key role in the bearing’s load transmission and frictional behavior.

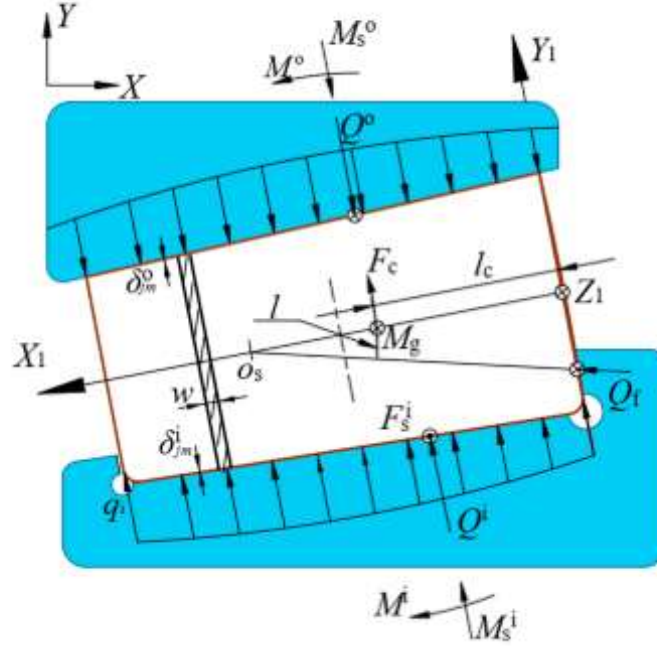


FIGURE 1.9 Roller & Raceway Contacts (W. Zhang & Li, 2025a)

1.5.2 Roller, Cage and Rib Contacts

Rib roller end contact geometry as shown in FIGURE 1.10 is crucial in TRBs because it governs the secondary forces and moments between the roller end, guiding rib, and cage, as reported in (W. Zhang & Li, 2025b), especially under high speed, light load, or poor lubrication where it can cause instability and failure.

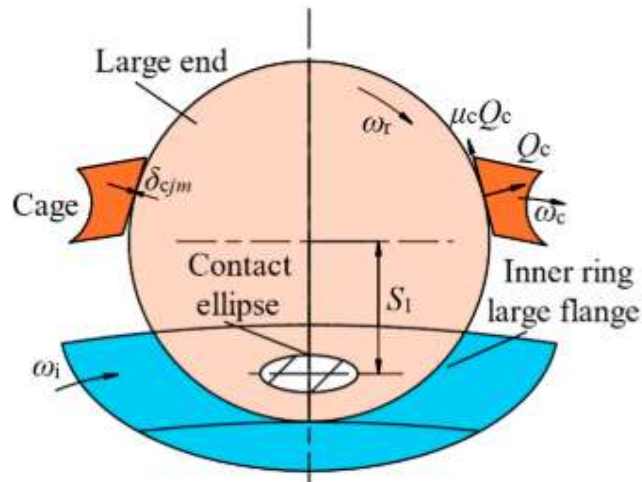


FIGURE 1.10 Roller, Cage and Rib Contacts (W. Zhang & Li, 2025b)

According to (Witte & Hill, 1987), the rib face is conical and the roller end is spherical, forming an initial point contact that becomes an elliptical contact area under load. The ellipse size and position depend on load, material elasticity, and internal geometry. Increasing the spherical roller end radius reduces stress but elongates the ellipse, risking its extension beyond the rib–roller contact zone. Thus, geometry must be optimized to maintain acceptable stress and keep the ellipse within the common rib roller region.

1.6 Numbering System for TRBs

The identification code used for TRBs as shown in FIGURE 1.11 follows a standardized structure that helps classify bearings based on their geometry, sizing, and design characteristics. The example shown in the figure uses a four-section numeric code, where

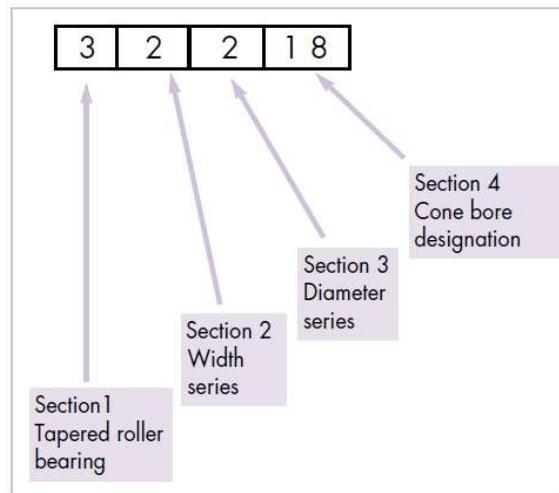


FIGURE 1.11 TRBs Numbering System (*GlobalSpec - Industrial Products & Supplier Catalog Search*, n.d.)

each segment provides specific information about the bearing. The first digit denotes the bearing type, where “3” signifies a TRB. The second digit represents the width series, which indicates the overall cross-sectional width of the bearing and reflects its capability to carry radial and axial loads. The third digit corresponds to the diameter series, describing the relative outer diameter for a given bore size and helping determine the bearing’s load-carrying robustness. The final two digits form the cone bore designation, which specifies the inner diameter of the bearing cone. Together, these sections offer a clear and systematic method for classifying TRBs, enabling accurate selection and comparison based on their dimensional and performance requirements.

1.7 Characteristics of TRBs

The performance and reliability of TRBs are governed by several important characteristics related to their geometry, load-carrying capability, operating conditions, and service life. These characteristics influence bearing efficiency, frictional behaviour, temperature rise, stiffness, and durability under combined radial and axial loads. Understanding these parameters is essential for proper bearing selection, design optimization, and performance evaluation in automotive and industrial applications. The major characteristics of TRBs are discussed as following.

1.7.1 Contact Angle

The contact angle refers to the the angle formed between the roller contact line and a plane normal to the bearing axis. It influences the axial load capacity—higher angles support greater thrust loads. In TRBs, the contact angle is determined by the geometry of the inner and outer raceways and directly affects load distribution between the rollers and raceways. Bearings with larger contact angles are preferred in applications subjected to significant axial loads, while smaller contact angles are suitable for predominantly radial loading conditions.

1.7.2 Preload

Preload is a small axial force applied to eliminate internal clearance and improve stiffness. Proper preload reduces vibration, improves rigidity, and minimizes deflection, but excessive preload increases friction and heat. In precision applications such as automotive wheel hubs and machine tool spindles, preload helps maintain accurate shaft positioning and improves operating stability. However, careful adjustment is necessary because over-preloading can accelerate wear and reduce bearing service life.

1.7.3 Endplay

Endplay represents the axial looseness or freedom of movement within the bearing. It is the opposite of preload and must be controlled precisely to avoid misalignment or excessive roller movement. Appropriate endplay allows for thermal expansion of bearing components during operation and prevents excessive stress generation. Incorrect endplay settings may result in vibration, noise, uneven load distribution, and reduced bearing performance.

1.7.4 Basic Dynamic Load Rating (C)

Here C indicates the bearing's capability to endure a given load for one million revolutions with acceptable fatigue life. It is an significant parameter in bearing selection and design. The rating is determined according to standardized testing procedures and is used to estimate bearing life under dynamic operating conditions. A higher dynamic load rating generally indicates a greater ability to withstand cyclic loading and achieve longer service life.

1.7.5 Basic Static Load Rating (C_0)

This rating represents the maximum stationary load that a bearing can sustain without inducing permanent deformation of the rollers or raceways. Static load rating is particularly important in applications subjected to shock loads, heavy static loads, or low-speed operation. Exceeding the static load rating may lead to indentations on raceway surfaces, resulting in increased vibration, noise, and reduced bearing reliability.

1.7.6 Frictional Torque

Frictional torque represents the resistance encountered when the bearing rotates. It is influenced by factors such as lubrication, roller geometry, preload, and surface finish. Reducing torque is critical for improving system efficiency. Excessive frictional torque increases power consumption and heat generation, which can degrade lubricant properties and shorten bearing life. Therefore, minimizing frictional torque is an important objective in the design and optimization of TRBs, especially in automotive and high-speed industrial applications.

1.7.7 Misalignment

Misalignment refers to the deviation of the shaft from the bearing centreline. TRBs tolerate limited misalignment; excessive levels lead to uneven load distribution and premature wear. Misalignment may arise from shaft deflection, housing inaccuracies, assembly errors, or operational loads. Prolonged operation under misaligned conditions can increase contact stresses, friction, vibration, and temperature, ultimately reducing bearing performance and fatigue life.

1.7.8 Fatigue Life (L10 Life)

Fatigue life refers the number of revolutions completed by 90% identical bearings can operate without failure. It is a fundamental parameter in predicting bearing longevity. The L10 life is influenced by operating load, rotational speed, lubrication conditions, contamination levels, and material properties. Bearing manufacturers commonly use L10 life calculations to assist engineers in selecting bearings capable of meeting the required reliability and service-life expectations of a given application.

1.7.9 Operating Temperature

Operating temperature is a critical performance characteristic of TRBs. Temperature rise occurs due to frictional losses, lubricant churning, and load-induced contact stresses. Excessive operating temperature can reduce lubricant viscosity, accelerate material degradation, and negatively affect bearing life. Therefore, monitoring and controlling BT is essential for ensuring reliable operation, particularly during high load with high speed operations.

1.7.10 Lubrication Performance

Lubrication is essential for minimizing direct contact between the rolling elements and raceways. Adequate lubrication lowers friction, wear, and heat generation while protecting the bearing against corrosion and contaminants. The selection of appropriate lubricant type, viscosity, and lubrication method significantly affects bearing efficiency, durability, and service life. Inadequate lubrication is among the most common causes of early bearing failure.

1.7.11 Bearing Stiffness

Bearing stiffness is the ability of a bearing to resist elastic deformation under applied loads. High stiffness is desirable in applications requiring accurate shaft positioning and minimal deflection. TRBs generally exhibit greater stiffness than many other rolling-element bearings because of their line-contact geometry and adjustable preload capability. Increased stiffness contributes to improved system precision, vibration control, and load-carrying performance.

1.8 Conclusion

This chapter established the overall background of the work and significance of RCBs, highlighting their historical development, essential components, classification, and relevance in modern engineering applications. The discussion of market trends further emphasized the continued importance and wide application of RCBs in various industrial sectors. The chapter subsequently introduced TRBs, with emphasis on their ability to support combined radial and axial loads and their significance in automotive and industrial applications. The associated research problem was identified, particularly the influence of frictional torque and the resulting heat generation on bearing performance, operating temperature, efficiency, and service life. The essential components of TRBs, their geometry, numbering system, and operating characteristics were also discussed to provide a fundamental understanding of their construction and functional behaviour. These aspects establish the technical basis for investigating the factors affecting frictional torque in TRBs. Overall, this chapter provides the necessary background and technical foundation for the present research and establishes the motivation for the experimental investigation and analysis of friction torque in TRBs presented in the subsequent chapters.

CHAPTER 2

Literature Review

2.1 Introduction

TRBs play a vital role in modern mechanical systems, particularly in applications where combined radial and axial loads supported efficiently. Their conical rolling elements and angled raceways enable them to withstand heavy and fluctuating loads, making them suitable for automotive, heavy machinery, railways, aerospace, and other high-performance sectors. This chapter presents a review of lubrication effects and friction torque in TRBs. It discusses previous studies under different geometrical and operating conditions, identifies the major sources and resistance mechanisms contributing to friction torque. It reviews the theoretical modelling approaches used to analyze friction torque and various experimental setups for frictional torque measurements in TRBs.

2.2 Effects of Lubrications in TRBs

Lubrication plays a significant role in determining the performance, reliability, and operational life of TRBs. Lubricant formulations have historically been optimized to prevent galling and wear in highly stressed gear contacts; however, less emphasis has been placed on understanding their influence on rolling contact fatigue. Nixon and Zantopulos emphasized that standardized laboratory tests, although widely used to differentiate lubricant performance, often overlook fatigue-related behavior that can significantly affect the long-term durability of bearing components. Their findings suggest that the industry must broaden its view of lubricant formulation to include fatigue resistance as a critical performance factor, especially for TRBs where rolling contact fatigue governs service life (Nixon & Zantopulos, 1995).

High-speed TRB applications introduce additional lubrication challenges due to rapidly increasing temperatures and frictional losses. Parker and Signer demonstrated through extensive high-speed testing that the method of lubricant delivery has a profound impact on bearing performance. Their study revealed that cone-rib lubrication—where lubricant is

supplied directly to the rib–roller end interface through drilled holes—significantly reduces cone-face temperature and power loss at rotational speeds reaching 15,000 rpm. They further reported that increasing shaft speed raises BT and power loss, while increasing the lubricant flow rate helps reduce thermal stress. This signifies the importance of effective lubrication pathways in mitigating thermal instability and minimizing energy losses in high-speed TRBs, particularly where centrifugal effects tend to limit lubricant availability (Parker & Signer, 1978).

In addition to lubrication supply, the cleanliness of the lubricant has a major impact on bearing wear behavior. Fitzsimmons and Clevenger found that TRB wear accelerates significantly in the presence of contaminants such as dirt and metallic debris. They identified that the concentration, size, and hardness of contaminant particles are key factors influencing wear progression. Their research also showed that lubricant viscosity, type, and duration of operation affect particle entrainment and damage mechanisms. Even under controlled conditions, some contamination is unavoidable, and its effects can lead to dimensional changes, surface degradation, and premature failure if not properly managed. This emphasizes the importance of effective filtration, proper sealing, and regular oil condition monitoring (Fitzsimmons & Clevenger, 1977).

More recent studies have focused on the electrohydrodynamic lubrication (EHL) characteristics of TRBs, particularly the influence of roller and raceway free-end effects on pressure distribution, film formation, and deformation. Chen et al. introduced an advanced EHL model that accounts for material flexibility at free ends, providing a more accurate prediction of lubrication performance than traditional half-space assumptions. Their work showed that roller fillet length, fillet radius, and Lundberg profile crowning strongly influence the minimum film thickness and stress concentration. The model also demonstrated that axial forces applied at the roller large end significantly affect deformation patterns and lubrication behavior. These findings underline the sensitivity of TRB lubrication to geometric and micro-structural parameters, highlighting the need for optimized roller-profile design (Chen et al., 2023).

Lubrication also affects the dynamic behavior and kinematic motions of TRB components. Research by Wu et al. revealed that under grease lubrication, roller tilt and skew are strongly dependent on operating speed, preload, and grease rheology. Roller tilt is greater in the unloaded zone, while skew tends to be higher in the loaded zone. Increased rotational speed

intensifies tilt, skew, and slip, whereas higher grease viscosity reduces slip and skew but increases tilt. Temperature variations further influence grease rheology and, consequently, bearing dynamics. Their study establishes that lubrication is not merely a passive film between rolling surfaces but an active factor influencing cage stability, roller alignment, and dynamic response (Z. H. Wu et al., 2018).

A broader perspective on lubrication research trends in TRBs is provided by Khan et al., who systematically reviewed numerical and experimental studies related to lubrication in different TRB configurations. Their analysis indicated that lubrication significantly affects frictional behavior, thermal stability, fatigue life, dynamic characteristics, and hydraulic (churning and drag) losses. Despite substantial progress, they noted limited research on large TRBs (outer diameter >300 mm), commonly used in wind turbines, rail systems, and industrial gearboxes. Furthermore, investigations involving non-Newtonian lubricants in large-scale TRBs remain scarce. These gaps highlight future opportunities to advance lubrication modeling and experimental validation in heavy-duty bearing applications (Ishaq Khan et al., 2025).

2.3 Review of FT in TRBs under Various Geometrical Parameters and Operating Conditions

In last few decades numerous work has been proposed to predict the the friction torque (FT) in TRBs. The early FT models were suggested by (Witte, 1973) and (Karna, 1974) for roller raceway and large rib roller end respectively. Later on many researcher had work on different friction models with different criteris, operating conditions and methodologies. The detailed summury of the review of the FT identification for TRBs are illustrated in TABLE 2.1.

TABLE 2.1 Summary of the Literature on Friction Torque in TRBs

Ref.	key considerations	Lubricants Oil /Grease	Speed (rpm)	Bearing Temp Y/ N	Axial Load (N)	Radial Load (N)	Software Model	Testing	Theoretical Model
(Witte, 1973)	roller and raceway contacts	oil	200 to 4000	✓	1000	2445-6890	✓		✓
(Karna, 1974)	rib and roller end	oil	100-1500	✓	3000			✓	✓
(Jamison et al., 1977)	roller skewing	oil	15 in/s		100 pounds			✓	
(Glöckner, 1983)	lubricants	oil	100		0-8000			✓	✓
(Gadallah & Dalmaz, 1984)	rib and roller end interactions	oil	500-1000		10-320			✓	✓
(Tsuneo Yamada, 1986)	rib roughness	oil	50	✓	3200			✓	✓
(S. Aihara, 1987)	inner diameter size	oil	1000-3000	✓				✓	✓
(Witte & Hill, 1987)	rib and roller end interactions	oil	200-2660		Preload	Preload		✓	✓
(Zhou & Hoeprich, 1991)	contaminated roller raceway and roller end rib contacts	oil	5000-30000					✓	✓
(S. Wang et al., 1993)	sever sliding conditions	oil	560			80000	✓	✓	✓
(Matsuyama et al., 1998)	roller and raceway interactions	oil	200-1000		1600-32000			✓	✓
(Matsuyama & Kamamoto, 2001)	roller and raceway interactions	oil	100-1500		2000-32000			✓	✓
(Houpert, 2002)	raceway length	oil	0-10000		13400			✓	✓
(SKF, 2003)	bearing characteristics	oil	--	--	--	--		✓	✓
(Paleu et al., 2004)	bearing geometry and lubricant flow	oil	500-3000		4000			✓	✓
(Blake & Truman, 2004)	combine loadings	oil, Grease	0-4000		1500-5500	0-16500		✓	✓
(Yang et al., 2008)	operating parameters and lubricant viscosity	Grease	0-1400		12000, 15000			✓	✓
(JTEKT, 2009)	application-based analysis	oil, Grease	500		40000	53000		✓	✓
(Cousseau et al., 2010)	lubricant characteristics	Different Grease, And Different oil	0 - 6000		7000			✓	✓

(MATSUYAMA, 2010)	bearing geometry and lubrication	oil	0-8000		No load	No Load		✓	✓
(Y. Li, 2014)	preloaded bearing under combined loading				544000	451000		✓	✓
(Alina et al., 2014)	parallel axis misalignment	oil	1000-5000		1000	3000		✓	✓
(Ailin & Jiugen, 2014)	temperature distribution and scuffing failure	Grease	5600	22-24	300			✓	✓
(Noguchi et al., 2014)	inner ring large flange surface	oil	500		10000			✓	✓
(Ai et al., 2015)	operating conditions and roller end geometry	Grease	1543 - 3000		25000	75000	✓		✓
(Lostado et al., 2015)	loading conditions and friction torque optimization	oil			-200	62500		✓	✓
(Wan et al., 2015)	romax based performance analysis	--			0-400		✓		✓
(V. Tong & Hong, 2015)	geometric error		1000		Preload 1500	500-1000	✓	✓	✓
(Liebrecht et al., 2015)	lubrication power losses	oil	1000-6000		0-1000 NM		✓	✓	✓
(Roda-casanova & Sanchez-marín, 2016)	bearing deflection	--	500		400-1400			✓	✓
(V. C. Tong & Hong, 2016)	angular misalignment	oil	500-3000		1000		✓		✓
(V. C. Tong & Hong, 2017)	load distribution and misalignment		500			25-250	✓	✓	✓
(J. Xu & Sheng, 2017)	preload with no-load friction torque	oil	2000-4500		Preload		✓	✓	✓
(Hammami et al., 2018)	lubricant viscosity and formulation	oil	9500		4000-7000			✓	✓
(J. Liu et al., 2018)	radial load, raceway velocity, and roundness error	oil	500-3000			1000-4000		✓	✓
(Laderou et al., 2019)	thermal effects on bearing preload	oil	1000-5000		Preload			✓	✓
(T. Xu et al., 2019)	misalignmen, pre-deformation, and radial clearance	oil	0-3500		4000	5000	✓		✓

(C. Zhang et al., 2019)	skewing of roller	Dry lubricated	60		Preload	Preload		✓	✓
(H. Wu et al., 2019)	vibration characteristics and raceway waviness	oil	1800		15000		✓		✓
(Y. Liu et al., 2020)	defect on roller raceway	oil	1500		1500	2500		✓	✓
(J. Liu et al., 2020)	waviness effect	Gas –oil mixture	500-2000		20			✓	✓
(Cruz et al., 2021)	bearing dimensions, lubricant viscosity, and operating speed	oil	2000 - 2500		Preload	No Load		✓	✓
(Mármol Fernández et al., 2021)	optimized bearing geometry under misalignment	oil	2000, 4000		5000	1000	✓	✓	✓
(Fujikake, 2021)	optimized cage and roller geometry	oil	2000, 5000		3000			✓	✓
(Y. Wang & Zhang, 2023)	surface texturing under starved lubrication	starved lubrication			1500	1000		✓	✓
(Y. Wang et al., 2022)	surface texturing and tribological performance	starved lubrication			40000			✓	✓
(Y. Liu et al., 2022)	geometric homogeneity	oil	1000		10000		✓		✓
(L. Liu et al., 2022)	raceway surface finish and ring convexity	oil	100,200		500 - 10000			✓	✓
(Wrzochal et al., 2022)	rolling raceway	oil	50,150, 250		11000		✓	✓	✓
(Toda, 2022)	high-speed oil bath lubrication	oil	2418	✓	16000	53000	✓	✓	
(Kelley et al., 2022)	micro-textured bearing surfaces	Grease	10		40000			✓	✓
(Manjunath et al., 2023)	EHL based rolling resistance analysis	oil	220 to 2200 rpm		2500 – 45000			✓	✓
(Lee & An, 2024a)	roller geometries with material variabilities	oil			✓			✓	✓
(W. Zhang & Li, 2025b)	roller end and rib curvature ratio	oil / Grease	6000		12000	24000		✓	✓

(Hu et al., 2025)	tri-axial rock-breaking loads and preload	Grease	low		Preload	Preload		✓	✓
(Xiong et al., 2026)	preload effect, ball friction	oil	500			9217		✓	✓

2.4 Main Sources of Friction Torque in TRBs

Palmgren first recognized the effect of lubricant viscosity in bearing torque prediction (Palmgren A, 1959), and with the advancement of EHL theory, studies by Van C. Tong and Harris and Van C. Tong and Zaretsky established the influence of EHL-based lubricant film thickness on rolling-element bearing friction (Harris & Kotzalas, 2006b)(V. C. Tong & Hong, 2014). Early predictive models such as Witte considered only rolling contacts, which correlated well under moderate conditions but later showed limitations under high-load, low-speed, and low-viscosity lubrication (Witte, 1973). Subsequent analyses by HcClintock et al. and Witte revealed that rib–roller end contact friction becomes significant under such conditions, highlighting that torque in TRBs originates primarily from two surfaces: the rolling raceway contacts and the rib–roller end contacts, with rolling contact dominating in most normal operating conditions (HcClintock & Ralph, 1985) (Witte & Hill, 1987).

Building further on the mechanisms governing torque generation, (S. Aihara, 1987) provided a unified analytical breakdown in which total frictional resistance consists of four main contributors: (1) rolling friction in the roller–raceway contacts (elastic hysteresis + viscous EHL resistance), (2) sliding friction at the rib–roller end interface, (3) roller–cage pocket sliding, and (4) viscous drag of the lubricant. He also noted that at normal speeds, cage friction and lubricant drag are minor, making rolling friction and rib-end sliding the primary torque sources, an observation consistent with (Witte & Hill, 1987).

I. Bercea et al. expanded these mechanisms by identifying viscous film friction, TRB resistance, micro-asperity traction, hydrodynamic resistance, spin friction, and air–lubricant mixture resistance as the main friction contributors (Bercea et al., 1996), while SKF General Catalogue summarized them into two broad categories: rolling-element/raceway contact friction and lubricant-flow-induced friction. Both energy dissipation mechanisms are highly influenced by the capability of the grease to maintain an effective lubricant film between the bearing surfaces (SKF, 2003).

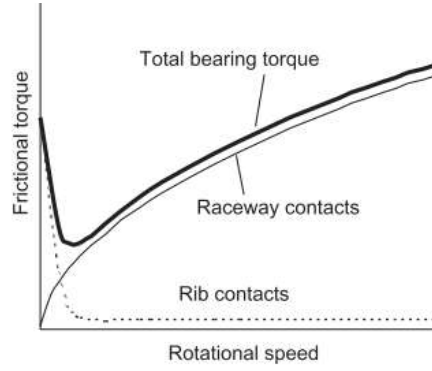


FIGURE 2.1 Frictional Torque of TRB (Matsuyama & Kamamoto, 2001)

Matsuyama et al. reported that the relationship between frictional torque and rotational speed of TRBs exhibits clear speed-dependent behavior as shown in FIGURE 2.1 at low speeds, sliding frictional resistance at the roller end and rib contacts is the dominant source of torque, while with increasing speed, the rib-contact torque decreases and the viscous rolling resistance within the raceway contacts increases, causing viscous rolling resistance to become the major contributor to total bearing torque at practical operating speeds (Matsuyama & Kamamoto, 2001).

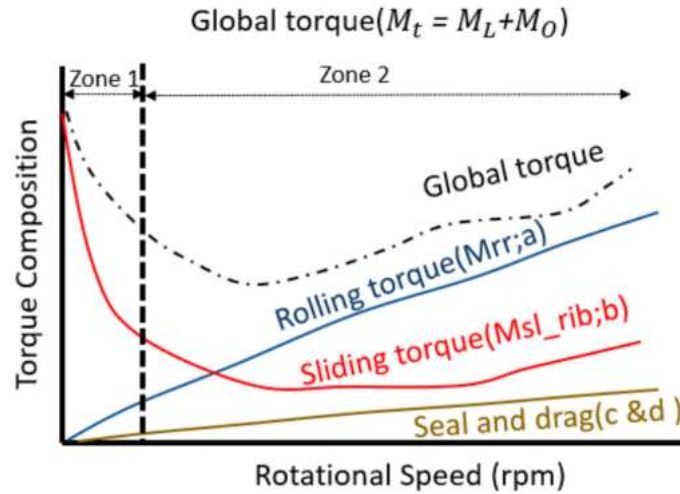


FIGURE 2.2 TRBs Torque Composition (Matsuyama & Kamamoto, 2001)

M. Manjunath et al. presented the FT in TRBs is commonly categorized into (a) load dependent components - tractive rolling torque M_{rr} and rib sliding torque $M_{sl_{rib}}$ driven largely by viscous contact losses and (b) load independent components - cage friction M_{cage} and lubricant drag M_{drag} as shown in FIGURE 2.2 which remain relatively small at moderate speeds as said Aihara (S. Aihara, 1987) (Manjunath et al., 2023). Thus, rolling raceway

viscous resistance and rib roller end sliding friction are consistently identified by all authors as the dominant contributors to TRB friction torque under most operating conditions.

2.5 Frictional Torque Components and Resistance Mechanisms in TRBs

2.5.1 Rolling Resistance at the Roller–Raceway Contact

Rolling resistance between the roller and raceway is a significant contributor to frictional torque in TRBs, particularly under moderate to high speeds. This resistance primarily arises from elastohydrodynamic lubrication (EHL) effects, elastic deformation of the contacting surfaces, lubricant shear, and micro-slip within the rolling contact. Under full-film lubrication conditions, several studies have shown that roller–raceway rolling resistance dominates the overall bearing friction behavior (SKF, 2017; Witte & Hill, 1987).

EHL-based analytical and experimental studies have established the fundamental mechanisms governing roller–raceway rolling resistance. Matsuyama et al. demonstrated that lubricant viscosity and rolling speed are the dominant parameters influencing viscous rolling resistance, while load effects are comparatively small under fully flooded lubrication conditions. Simplified analytical expressions for frictional torque were proposed and validated experimentally (MATSUYAMA, 2010; Matsuyama et al., 1998; Matsuyama & Kamamoto, 2001). Zhou et al. further developed an analytical torque model incorporating EHL pressure distribution, surface roughness, and thermal effects, confirming that roller–raceway rolling resistance constitutes a major portion of the total bearing friction torque (Zhou & Hoeprich, 1991).

K. Yan et al., J. Jurko et al. and L. Liu et al. presented that raceway geometry and surface integrity strongly influence rolling resistance. Studies on raceway convexity and micro-geometry showed that appropriate convexity improves pressure distribution and reduces rolling resistance, whereas excessive convexity increases contact stress (Jurko et al., 2016; L. Liu et al., 2022; Yan et al., 2015). H Saruhan et al., H. Wu et al. and Y Liu et al. were found that surface waviness and raceway defects to cause localized pressure concentrations and uneven load distribution, significantly affecting rolling resistance and torque fluctuations during operation (Y. Liu et al., 2020; Saruhan et al., 2014; H. Wu et al., 2019).

Experimental investigations have focused on isolating roller–raceway friction torque from total bearing torque. G. Cai et al. proposed an energy-based experimental method to decouple roller–raceway rolling resistance under heavy radial loads, enabling accurate FT measurement without modifying bearing structure (Cai & Hou, 2024). Earlier experimental studies also highlighted the contribution of raceway rolling resistance under various lubrication regimes (Witte & Hill, 1987).

M. Gärtner et al. found material and lubrication effects further govern rolling resistance behavior. Advanced raceway materials and surface treatments were shown to enhance lubricant film stability and reduce friction under extreme operating conditions (Gärtner et al., 2023). Lubrication regime transitions, lubricant starvation, and surface texturing were found to significantly influence rolling resistance, particularly at higher speeds (Da Silva et al., 2015; SKF, 2017; Y. Wang & Zhang, 2023).

Overall, existing literature confirms that roller raceway rolling resistance in TRBs is primarily controlled by EHL lubrication behavior, raceway geometry, surface integrity, material properties, and operating conditions such as speed and load. Effective control of these parameters is essential for minimizing frictional torque and improving bearing efficiency and reliability.

2.5.2 Sliding Friction Between Roller End and Rib

Sliding friction between the roller end face and the guiding rib of TRBs has long been recognized as a major contributor to frictional torque, particularly under axial and combined loading conditions. Early analytical and experimental investigations highlighted that, unlike raceway contacts, the rib–roller end contact operates under inherent sliding due to bearing kinematics. C. L. Karna analytically studied the coefficient of friction at the large rib–roller end contact and demonstrated that inertial forces have negligible influence at normal bearing speeds, while sliding speed, Hertzian contact stress, and lubrication condition strongly govern rib torque (Karna, 1974). Experimental results confirmed that under hydrodynamic lubrication, rib torque is significantly reduced. W. E. Jamison et al. further emphasized that the geometric and kinematic complexity of TRBs imposes stringent demands on rib geometry and manufacturing tolerances, as rib contacts directly affect bearing performance and torque generation (Jamison et al., 1977).

Subsequent studies focused on quantifying the contribution of rib–roller end contact to overall bearing torque. Witte identified rib–roller end sliding as one of the two dominant sources of torque in TRBs, alongside raceway contacts, and demonstrated that rib torque becomes predominant under certain operating conditions (Witte & Hill, 1987). Tsuneo similarly reported that TRBs generate higher frictional torque than ball or cylindrical roller bearings primarily due to sliding friction at the rib area. The author highlighted that reducing rib friction can lead to significant improvements in operating temperature, preload control, seizure resistance, and overall bearing reliability, particularly in low-speed applications where rib contact effects are dominant (Tsuneo Yamada, 1986).

With advances in lubrication theory, several researchers examined the rib–roller end contact under hydrodynamic and elastohydrodynamic lubrication (EHL) regimes. Gadallah et al. developed a hydrodynamic model for the lubricated rib–roller end contact and validated it experimentally using optical interferometry, showing good agreement between predicted and measured film thickness and traction forces (Gadallah & Dalmaz, 1984). Extensions into the EHL regime revealed deviations from classical fully flooded assumptions due to side-flow effects. Zang et al. provided full numerical EHL solutions for the elliptical rib–roller end contact, demonstrating that elastic deformation plays a significant role under practical loads and that curvature ratios and contact location significantly influence minimum film thickness and friction (Z. Zhang et al., 1988). F. Colin et al. further extended this work by analyzing starved EHL conditions, showing that oil supply, contact geometry, speed, and load have a strong influence on traction forces and frictional moments at the rib contact (Colin et al., 1998).

More recent research has increasingly emphasized the influence of rib and roller-end geometry on friction, wear, and energy efficiency. Wirsching et al. employed numerical EHL simulations combined with design of experiments and optimization algorithms to study the macro-geometry of the rib–roller end pairing. Their results revealed a trade-off between load-carrying capacity and friction losses, showing that spherical or toroidal roller-end geometries paired with tapered ribs are advantageous for minimizing friction, while other combinations favor higher lubricant film thickness and load capacity (Wirsching et al., 2021). Wenhui et al. further refined geometric optimization by evaluating different spherical base curvature radii under combined loading conditions, demonstrating that smaller

curvature ratios reduce FT, while intermediate values provide a balance between low friction and operational precision (W. Zhang & Li, 2025b).

In parallel, wear behavior at the rib–roller end interface has been investigated due to its strong dependence on sliding friction and lubrication. Klebanov et al. developed detailed wear rate prediction methods based on Archard’s law, incorporating dynamic loading, sliding speed, oil film thickness, and finite element contact pressure analysis. The study showed that rib geometry plays a crucial role in wear reduction, with toroidal concave rib surfaces reducing wear rates significantly compared to tapered surfaces. These findings highlight the close relationship between frictional behavior, lubrication regime, and long-term durability of rib contacts (Klebanov et al., 2022).

Recent studies have also addressed rib–roller end friction under realistic combined loading and dynamic conditions. Xin Bin et al. proposed an improved friction moment calculation method capable of modeling dry, boundary, mixed, and EHL lubrication regimes under combined axial and radial loads, overcoming limitations of simple empirical friction coefficients (X. Bin Li et al., 2024). Tengfei et al. incorporated rib–roller end sliding friction into a comprehensive FT model for double-row TRBs, showing that preload, clearance, and misalignment significantly affect rib-related torque components (T. Xu et al., 2019). W. Patrick et al. further validated multibody simulation approaches that explicitly include rib–roller end friction, demonstrating good agreement between simulated and measured FT across varying speeds, loads, and temperatures (Wingerts Zahn et al., 2023).

Additionally, rib contact condition has been shown to influence bearing vibration and fault behavior. Jing Liu et al. demonstrated that localized surface faults on the inner-ring rib significantly alter vibration characteristics due to non-Hertzian rib–roller interactions, particularly under combined loads (J. Liu et al., 2017). Parker et al. (1978) experimentally showed that targeted lubrication of the cone-rib contact significantly reduces temperatures and power loss at high speeds, confirming the critical role of rib lubrication in controlling frictional losses (Parker & Signer, 1978). Recent probabilistic and parametric studies, such as that by S. Lee et al. further indicated that rib angle and related geometric parameters have a strong statistical influence on FT, emphasizing the importance of rib–roller end design in modern low-torque bearing applications (Lee & An, 2024b).

2.5.3 Lubrication Resistance and Viscous Drag Losses

Lubrication resistance and viscous drag losses arise from lubricant shear, displacement, and churning within TRBs and become more pronounced at higher rotational speeds and with increased lubricant quantity. Compared with rolling resistance at the roller–raceway contact and sliding friction at the roller end–rib interface, these losses are generally reported to be of secondary importance under low to moderate speeds and high-load conditions typical of many TRB applications. Nevertheless, they can contribute noticeably to the total frictional torque when lubrication conditions are not optimized.

Liebrecht et al. reported that drag and churning losses depend strongly on lubricant viscosity, oil level, bearing type, and operating speed. Although bearing manufacturers provide empirical models to estimate these losses, comparisons with experimental results revealed deviations, indicating that lubricant flow behaviour inside the bearing is complex and not fully captured by simplified analytical formulations. Experimental and numerical studies showed that increasing oil quantity leads to higher FT due to enhanced churning effects, even though these losses remain lower than contact-related friction components under comparable operating conditions (Liebrecht et al., 2015).

The influence of viscous drag losses on BT has been particularly observed in grease-lubricated bearings. Siyuan Ai et al. demonstrated that higher grease filling ratios and rotational speeds result in increased temperature rise, primarily due to enhanced viscous shear and drag. However, the study also indicated that these losses are strongly dependent on lubrication quantity and geometry, and can be controlled through optimized grease filling and roller-end geometry, thereby limiting their overall contribution relative to rib–roller sliding friction (Ai et al., 2015).

Further insight into lubrication-induced losses was provided by Liebrecht et al. through combined experimental measurements and CFD simulations, which enabled separation of drag and churning losses and the development of empirical relations for hydraulic power loss estimation (Liebrecht, J.; Si, X.; Sauer, B.; Schwarze, 2016; Liebrecht et al., 2015). Liebrecht et al. (2015) and Marchesse et al. (2019) additionally showed that optimizing oil-circulation flow rates significantly reduces drag losses in global torque measurements, confirming that viscous losses can be effectively minimized through proper lubrication management (Liebrecht et al., 2015) (Marchesse et al., 2019).

In summary, the literature indicates that lubrication resistance and viscous drag losses in TRBs are generally smaller than rolling resistance and rib–roller end sliding friction, particularly under high-load and low-to-moderate speed conditions. However, under high-speed operation or excessive lubricant supply, these losses can become significant and contribute to increased frictional torque and BT, emphasizing the importance of optimized lubrication strategies.

2.6 TRBs Friction Torque Theoretical Modeling

Theoretical modelling of friction torque in TRBs has been extensively studied by many researchers to understand the influence of bearing geometry, lubrication conditions, load, and operating speed on bearing performance. Various analytical and semi empirical models have been proposed to predict friction torque by considering different frictional components and contact mechanisms. These models provide a theoretical basis for estimating friction torque and serve as valuable tools for bearing design, performance evaluation, and optimization. The principal theoretical models and their corresponding equations reported by various researchers are consolidated in TABLE 2.2.

Witte developed one of the earliest comprehensive models for predicting the operating torque of TRBs by integrating elastohydrodynamic (EHD) contact principles with bearing geometry and lubrication characteristics. FIGURE 2.3 shows the primary load distribution on rollers in TRBs. The study explained that FT arises primarily from the skewed pressure distribution as shown in FIGURE 2.4 in the roller–race contacts, which shifts the centroid of pressure and generates a moment on each roller. Using dimensional analysis, the key EHD variables load per unit length, rolling speed, lubricant viscosity, and pressure viscosity coefficient were related to roller forces, resulting in a general torque expression dependent on bearing geometry through a defined G-factor, representing the heat-generation capability of the bearing. The model was later extended for radial and combined loading conditions as shown in Equation 2.1 through the introduction of an equivalent thrust-load factor producing the generalized torque expression in Equation 2.2. Overall, the FT model provided a practical and experimentally validated framework to estimate operating torque and associated heat generation under varying loads, speeds, and lubrication conditions (Witte, 1973).

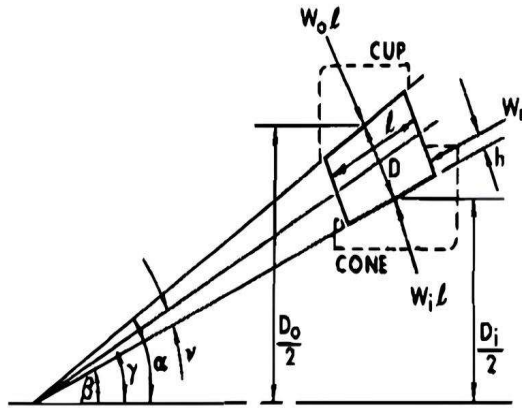


FIGURE 2.3 Load Reaction on Rollers
(Witte, 1973)

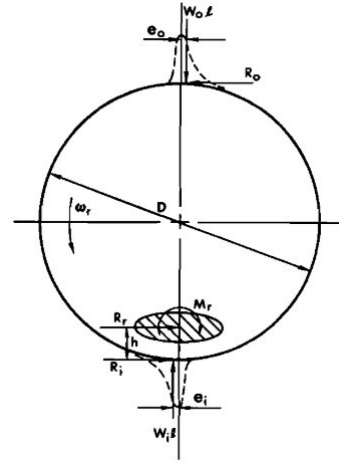


FIGURE 2.4 Friction Forces at Roller End
(Witte, 1973)

Karna presented FT in TRBs has focused extensively on understanding the mechanical and tribological interactions at the rib - roller contact as shown in FIGURE 2.5, a critical region influencing bearing performance. Analytical studies have examined the dynamic equilibrium of rollers to evaluate how inertial forces affect rib reaction forces. These works consistently show that, under normal operating conditions, the dynamical increase in rib reaction due to inertia is minimal, though at high rotational speeds the centrifugal forces become significant and must be incorporated into rib-torque calculations. Mathematical models have been developed to describe rib torque as shown in Equation 2.3 including simplified expressions that facilitate engineering application (Karna, 1974).

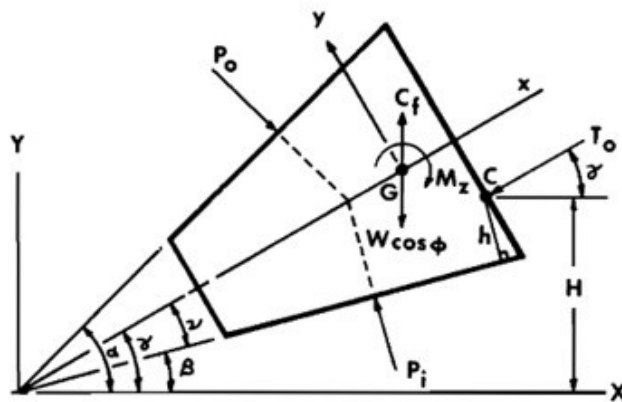


FIGURE 2.5 Forces Acting on TRBs Rollers (Karna, 1974)

Building on the limitations observed in earlier FT formulations, Aihara identified that conventional equations for estimating the running torque of TRBs (TRBs) often deviated

significantly from experimental measurements, particularly under axial loading conditions as shown in FIGURE 2.6. To address this discrepancy, a revised analytical approach was

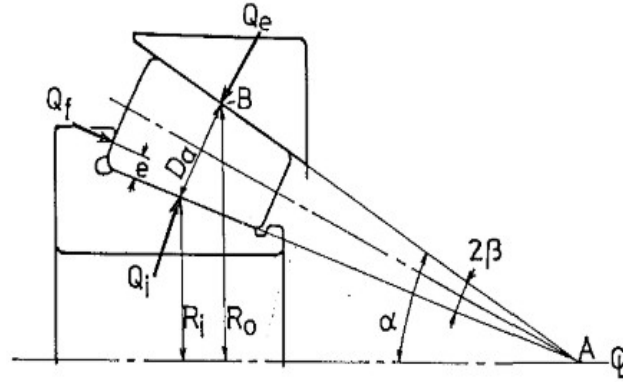


FIGURE 2.6 Forces on TRBs roller (S. Aihara, 1987)

developed by incorporating elastohydrodynamic lubrication (EHL) rolling resistance and EHL oil film thickness into the torque prediction model. The revised running torque equation as shown Equation 2.4 was compared against extensive experimental datasets, including results from nearly two hundred bearings covering bore sizes from 17 mm to 120 mm under standardized operating conditions (S. Aihara, 1987).

TABLE 2.2 TRBs Friction Torque Theoretical Modeling

Paper	Model	Friction Torque Equation	Equation No
(Witte, 1973)	Pure Thrust Load	$M = 1.1 \times 10^{-4} G (S\mu)^{1/2} (F_a)^{1/3}$	2.1
	Radial & Combined Loads	$M = 1.1 \times 10^{-4} G (S\mu)^{1/2} \left(\frac{f_T F_r}{K} \right)^{1/3}$	2.2
(Karna, 1974)	Total Rib Torque	$T_f(rib) = \mu H \left(2W_r \frac{\sin v}{\sin a} + nC_F \sin \beta \right)$	2.3

(S. Aihara, 1987)	Running Torque	$M = \frac{Z}{D_a} (R_0 M_i + R_i M_0) + e \cdot \mu_0 \cos \beta$ $\cdot F_a \exp(-1.8 \Lambda_r^{1.2})$ Where, $M_{i,0} = \left[\left(\frac{1.76 \times 10^2}{1 + 0.29 L^{0.78}} \right) \frac{1}{\alpha_0} (G U)^{0.658} W^{0.31} \right. \\ \left. \cdot R_e^2 \cdot 1 \right]_{i,0}$	2.4
(Zhou & Hoeprich, 1991)	Total Bearing Torque	$M_{BRG,total} = \sum_{j=1}^{z_r} \left[F_{0,j} (R_0 - R_{e,0} \cos(\alpha_i + \beta)) \right. \\ \left. + F_{i,j} (R_i + R_{e,i} \cos(\alpha_i + \beta)) \right. \\ \left. + F_{rib,j} \frac{h}{D_r} R_0 + M_{r,j} \frac{R_e}{D_r} \right]$	2.5
(Bercea et al., 1996)	Bearing Torque	(a) The bearing torque derived from inner Raceway contacts: $M_{cup} = M_i = \sum_{j=1}^z FS_{ij} \cdot RS_{ij} + \sum_{j=1}^z FR_{ij} \cdot RR_{ij} \\ + \sum_{j=1}^z FA_{ij} \cdot RA_{ij}$ (b) The bearing torque derived from outer Raceway contacts: $M_{cone} = M_o = \sum_{j=1}^z FS_{oj} \cdot RS_{oj} + \sum_{j=1}^z FR_{oj} \\ \cdot RR_{oj} + \sum_{j=1}^z FA_{oj} \cdot RA_{oj}$ (c) The bearing torque derived from the Rib-Roller End contacts: $M_{rib} = M_f = \sum_{j=1}^z (FT_{yf} + FA_f)_j \cdot R_f$	2.6

(Matsuyama et al., 1998)	Bearing Torque under fully flooded condition	$m_{BRG} = m_{race} + m_{rib}$ $m_{race} = \frac{z}{D_w} (r_o m_i + r_i m_o)$ $m_{rib} = \frac{z}{D_w} r_o c_o F_{rib}$ $m = 14.2 E' l R^2 \cdot U^{0.75} G^{-0.04} W^{0.08}$	2.7
(Matsuyama & Kamamoto, 2001)	frictional torque of the bearings lubricated in an oil bath	$M = 0.8 g_1 \phi_T U^{0.57} G^{-0.04} W^{0.22}$ $+ \mu e F_a \cos \gamma$	2.8
(Houpert, 2002)	Total Frictional Torque	$dT = 2. (FR_o + FR_i) \frac{R_o \cdot R_i}{d_m} + \frac{M_{rib} \cdot R_o}{D}$ $+ \frac{MER_i \cdot R_o + MER_o \cdot R_i}{D}$ $dT = dTR + dTRIB + dTER$	2.9
(SKF, 2003)	Total Frictional Torque	$M = M_{rr} + M_{sl} + M_{seal} + M_{drag}$	2.10
(Y. Liu et al., 2020)	Friction Torque	$T_{i,e} = \sum_{k=1}^{n_s} \left[\left(\frac{1.76 \times 10^2}{1 + 0.29 L^{0.78}} \right) \frac{1}{p_0} G U^{0.658} w_k^{0.31} R_k^2 \Delta l \right]$	2.11
(L. Liu et al., 2022)	Total Friction Torque	$M_i = r_i F_i + F_r \left(r_i + \frac{l}{2} \sin \alpha_i + i \cos \alpha_o \right)$ $+ m_i$ $M = z_l \cdot M_i$	2.12

Further advancements in TRB torque prediction were made by Zhou, who developed a comprehensive analytic torque model as shown in Equation 2.5, based on EHL and detailed micro macro contact mechanics. FIGURE 2.7 shows the different forces acting in TRBs and

raceway rolling resistance forces as presented in FIGURE 2.8, EHL-induced pressure and frictional forces generated at the rib roller end contact. Importantly, because the model evaluates the contact behavior of a single roller, it can be readily extended to pure axial, radial, or combined load scenarios, providing a versatile foundation for future torque analyses (Zhou & Hoeprich, 1991).

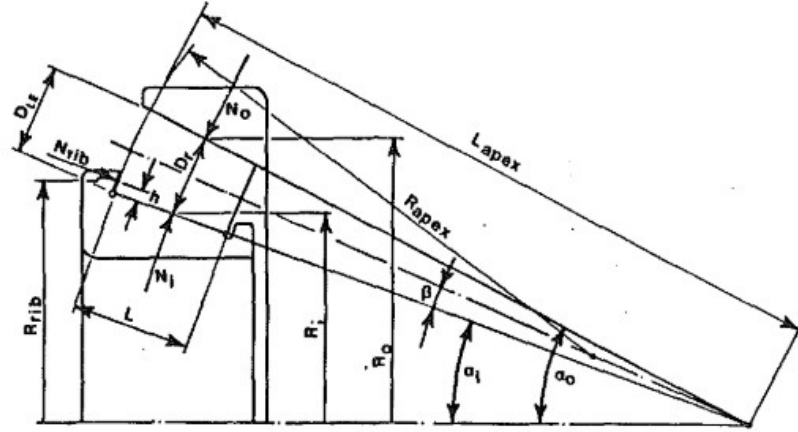


FIGURE 2.7 Forces on TRB Roller (Zhou & Hoeprich, 1991)

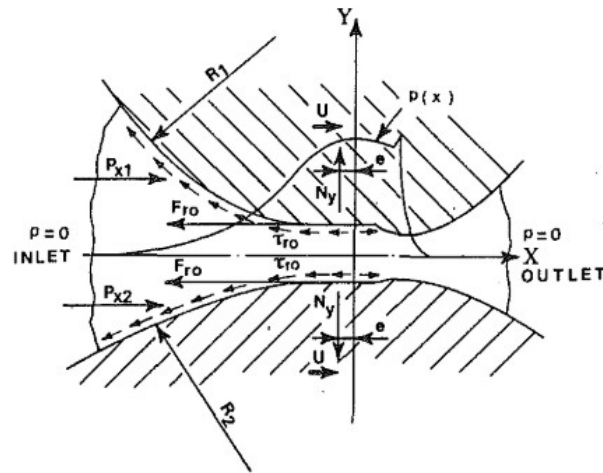


FIGURE 2.8 Rolling Resistant and Pressure Force Along With Normal Load Or Pure Rolling (Zhou & Hoeprich, 1991)

S. Wang et al presented a specialized torque and thermal generation model for TRBs operating under abnormal conditions involving partially or fully jammed rollers. By introducing an equivalent bearing mechanism, simplified differential equations as shown

Equation 2.6 were derived to describe frictional torque applied by the rollers due to inner raceway contacts and rib roller end contacts as shown in FIGURE 2.9 during excessive sliding and rolling components of friction. This work emphasized the critical impact of roller jamming on FT (S. Wang et al., 1993).

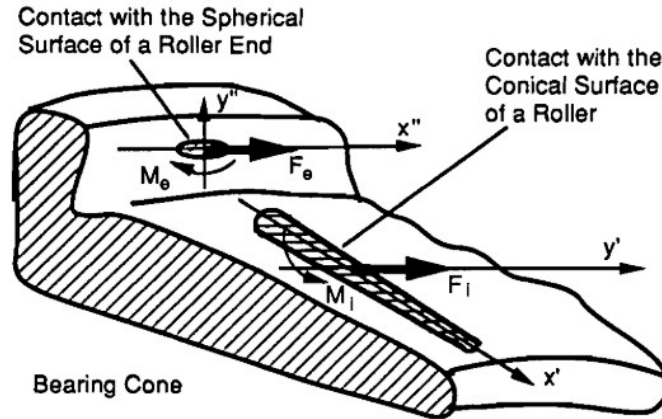


FIGURE 2.9 Contact Area Between Roller and Cone & Roller End and Rib (S. Wang et al., 1993)

Complementing earlier analytical and EHL-based formulations, Bercea introduced a refined FT model that captures both macro- and micro-contact frictional phenomena inherent in TRBs. The model evaluates FT generated at (a) the inner raceway contacts, (b) the outer raceway contacts, and (c) the rib–roller end contacts as shown Equation 2.6, thereby providing a complete breakdown of the dissipative mechanisms that influence bearing efficiency. Building upon these insights, Bercea formulated a comprehensive power-loss model capable of quantifying the individual contributions of distinct tribosystems within the bearing assembly. This granular approach to torque and power-loss prediction offers enhanced diagnostic capability and supports more accurate thermal, lubrication, and performance assessments for TRBs under diverse loading and speed conditions (Bercea et al., 1996).

Using Aihara's simplified torque decomposition, Matsuyama further advanced FT modelling in TRBs by developing a simplified expressions as shown Equation 2.7 for viscous rolling resistance in roller and raceway contacts based on full EHD line-contact analyses over a broad range of dimensionless lubrication parameters (Matsuyama et al., 1998).

Matsuyama extended earlier work on EHL-based torque prediction by performing full-line-contact EHL analyses across an even wider range of bearing operating conditions to derive a simplified viscous rolling-resistance formula for TRBs. This refined expression, valid under fully flooded lubrication, was incorporated into the raceway torque model and validated against experimentally measured frictional torque. The comparison showed good agreement between predicted and measured values, confirms the consistency of the simplified EHL formulation. In addition, Matsuyama proposed a new frictional torque expression for oil-bath lubrication to account for practical operating environments. Based on the combined theoretical and experimental investigation, a new raceway frictional torque equation was formulated and presented as Equation 2.8, providing an improved predictive tool for TRB torque estimation under varying lubrication conditions (Matsuyama & Kamamoto, 2001).

L. Houpert provided an analytical breakdown of the various components contributing to bearing torque and established a comparative framework between TRBs and ball bearings (BBs) under viscous lubrication conditions. By formulating explicit relationships for each torque component, the study emphasized how analytical expressions can support bearing design optimization and the selection of the most suitable bearing type for specific operating conditions. For TRBs, the torque expression was derived as Equation 2.9, enabling direct comparison with corresponding ball bearing formulations (Houpert, 2002).

The SKF introduced a comprehensive and practical framework for evaluating total bearing friction by identifying four primary contributors: rolling friction M_{rr} , sliding friction M_{sl} , seal friction M_{seal} , and drag-related losses M_{drag} . By expressing the total frictional moment as Equation 2.10, the model integrates all contact-level sources—including raceway and flange interactions—while also accounting for external effects such as lubricant churning and splashing. This contact-by-contact approach enables precise incorporation of design modifications, surface improvements, and updated bearing geometries, making the model adaptable to evolving SKF product designs (SKF, 2003).

L. Liu et al. developed an improved theoretical model as shown Equation 2.12 for predicting the FT of TRBs by incorporating the effects of raceway roughness and the convexity profiles of both inner and outer rings. The study evaluated roller–raceway friction under EHD lubrication and validated the model through controlled experiments that measured FT for bearings with varying convexities. The study further showed that FT rises substantially with

axial load and that the influence of convexity becomes more pronounced at higher loads, whereas reduced raceway roughness contributes to lower FT (L. Liu et al., 2022).

S. Lee et al. examined the FT characteristics of TRBs by focusing on the effects of roller geometry and material variability—areas that have received limited attention compared to lubrication and contact studies. The work combined theoretical calculations with experimental measurements for bearings operating under axial load. To assess the sensitivity of FT to geometric and material deviations, a Monte Carlo simulation was performed, revealing a left-skewed distribution and showing that the outer raceway half-angle and rib angle were the most influential parameters. The study demonstrated that the proposed approach can effectively identify key design factors affecting torque, offering valuable guidance for optimizing tapered-roller bearing performance (Lee & An, 2024a).

Wingerts Zahn et al. developed a parametric multi body dynamic model capable of accurately predicting both the FT and kinematics of TRBs. The model incorporates solid and lubricant friction, including key contact interactions such as roller and raceway, roller end with rib, and cage pocket contacts. The study concludes that the LaMBDA model provides highly reliable torque predictions due to its detailed contact and lubrication modeling. However, losses caused by lubricant flow outside the immediate contact zones—relevant in gearbox applications—are not yet included (Wingerts Zahn et al., 2023).

Manjunath et al. contributed to friction-model development in TRBs by experimentally examining the rolling resistance torque and the influence of thermal inlet shear under EHL conditions. The study clarified how different friction mechanisms dominate at different speeds—pressure-center shift in EHL contacts at high speeds and mixed-lubrication sliding at the roller–rib contact at low speeds. Using established friction sub-models from the literature, the author quantified rolling resistance contributions under varying speeds, loads, and temperatures, and demonstrated that velocity significantly influences EHL rolling friction, whereas material and load parameters have comparatively minor effects. A key modelling insight from this work is the incorporation of inlet shear-heating effects on the effective lubricant temperature, showing that thermal shear can reduce rolling torque by nearly 6–8% at high speeds. This provides an important refinement to existing TRB friction models by emphasizing the role of inlet thermal behaviour in predicting rolling friction accurately (Manjunath et al., 2023).

Wirsching et al. advanced friction modelling research for TRBs by focusing on the EHL roller end face and rib contact, which is a major contributor to secondary frictional losses. The study used Design of Experiments (DoE) to systematically vary geometric features of the roller and rib, and EHL contact simulations were employed to predict key tribological outputs such as friction, pressure, and lubricant film thickness. These results were further processed using a meta-model of optimal prognosis and optimized with evolutionary algorithms, enabling efficient prediction of low-friction geometric configurations. The work demonstrated that the friction behaviour of the roller face and rib contact is primarily governed by the fundamental geometric pairing and the radii, whereas eccentricity has limited influence. A clear trade-off was identified: geometries promoting low friction (e.g., spherical or toroidal roller end faces with large radii paired with conical ribs) may reduce load-carrying capacity, while configurations optimized for higher film thickness and load capacity require different radii combinations. Overall, this study provides a computationally efficient and machine-learning-assisted framework for optimizing TRB friction by tailoring roller–rib macro-geometry (Wirsching et al., 2021).

2.7 Experiment Setups for Friction Torque Measurements

Understanding FT in TRBs is essential for improving energy efficiency, predicting bearing performance, and optimizing lubrication strategies. The preceding sections have highlighted the influence of lubricant formulation, contamination, elastohydrodynamic film behavior, and dynamic roller kinematics on the overall frictional response of TRBs. While theoretical and numerical models provide valuable insights, accurate characterization of FT ultimately depends on reliable experimental measurement techniques. Researchers have therefore developed various test rigs and instrumentation approaches to capture torque behavior under controlled conditions that simulate real-world operating environments. These setups typically account for speed, load, temperature, lubrication supply conditions, and roller–rib interactions, allowing for systematic evaluation of frictional contributions from raceway contacts, rib–roller ends, and churning effects. This section reviews the different experimental setups employed in the literature, with emphasis on their design features, measurement principles, and suitability for analyzing FT in TRBs.

Witte conducted experimental work using a specially designed test rig and test bearing in which the rib was separated from the cone race. This design allowed the rib torque to be

measured independently of the rolling torque produced between the cup and cone race. Using this machine, shown in FIGURE 2.10 of his study, data were obtained for a wide range of bearing loads, operating speeds, and lubricant viscosities. Measurements were performed for both new and run-in bearings, and the results were used to determine the coefficient of friction under these varying conditions (Witte, 1973).

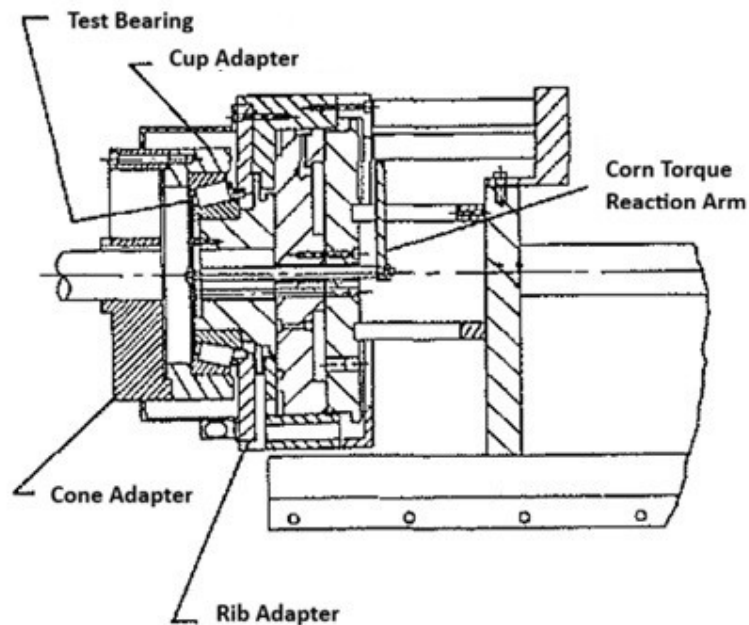


FIGURE 2.10 Bearing Rib Torque Measurement Machine (Witte, 1973)

Aihara measured the running torque of a single TRB using a specially designed test rig because direct measurement is not easy. The main components of the rig are shown in FIGURE 2.11 of his study. Each test bearing was installed at the end of a vertical shaft, while the axial load was transmitted through a hydrostatic thrust bearing, enabling the bearing housing to rotate freely. Lubrication was provided by a circulating oil system, and both the bearing and the lubricant were preheated to a predetermined temperature before each FT measurement. During operation, rotation of the inner ring generated frictional torque, which caused the bearing housing to rotate. The running torque was obtained by measuring the restraining force required to prevent the housing from turning, and this force was converted to torque. Torque data were recorded while slowly and continuously increasing the rotational speed up to 3300 rpm. The test bearings were randomly selected from those having bore diameters larger than 17 mm and outer ring diameters lesser than 240 mm, using different housings and shaft end fittings (S. Aihara, 1987).

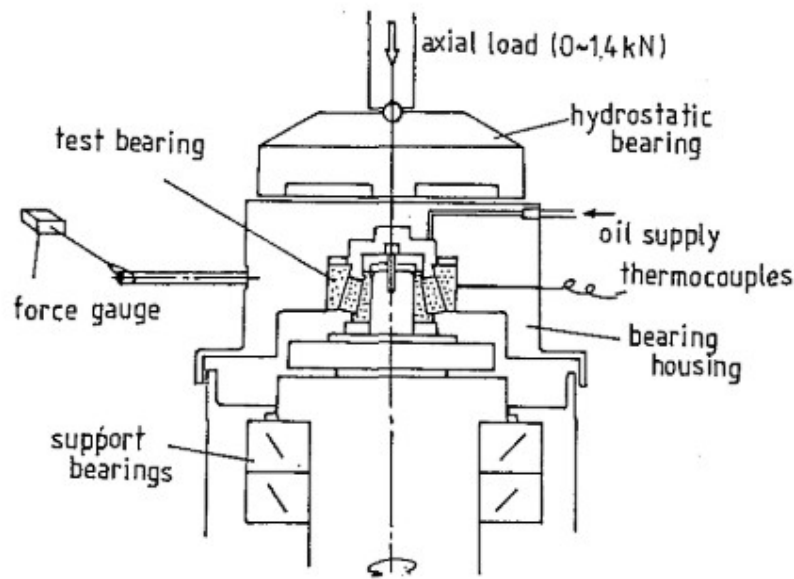


FIGURE 2.11 Running Torque Test Rig (S. Aihara, 1987)

Zhou developed a bearing torque test rig to independently measure the frictional torque of the cup race, cone race, and cone rib, thereby providing a better understanding of the torque characteristics of tapered roller bearings (TRBs). The proposed model accounted for both asperity contact and lubricant shear occurring at the roller end-rib interface. The schematic configuration of the test rig is presented in FIGURE 2.12 of the study. The test shaft carrying the bearing was supported by air journal bearings, while air thrust bearings applied the axial load and reaction load to the inner and outer races, respectively. The floating cup housing enabled FT measurement through the reaction force, and an air cylinder was used to generate the required axial load. The cone rib was isolated from the cone and connected through support arms instrumented with strain gauges to measure rib torque. Thermistors positioned near the rib, cone, and cup contact regions monitored the operating temperature, and all sensor signals were transmitted through a multi-channel rotary transformer. Thermal calibration of the sensors was performed after assembly to ensure accurate measurements. The experimental results indicated that asperity contact becomes significant under thin EHL film conditions. with model predictions demonstrated good agreement when thermal reduction factors were included, particularly at higher speeds and with more viscous lubricants (Zhou & Hoeprich, 1991).

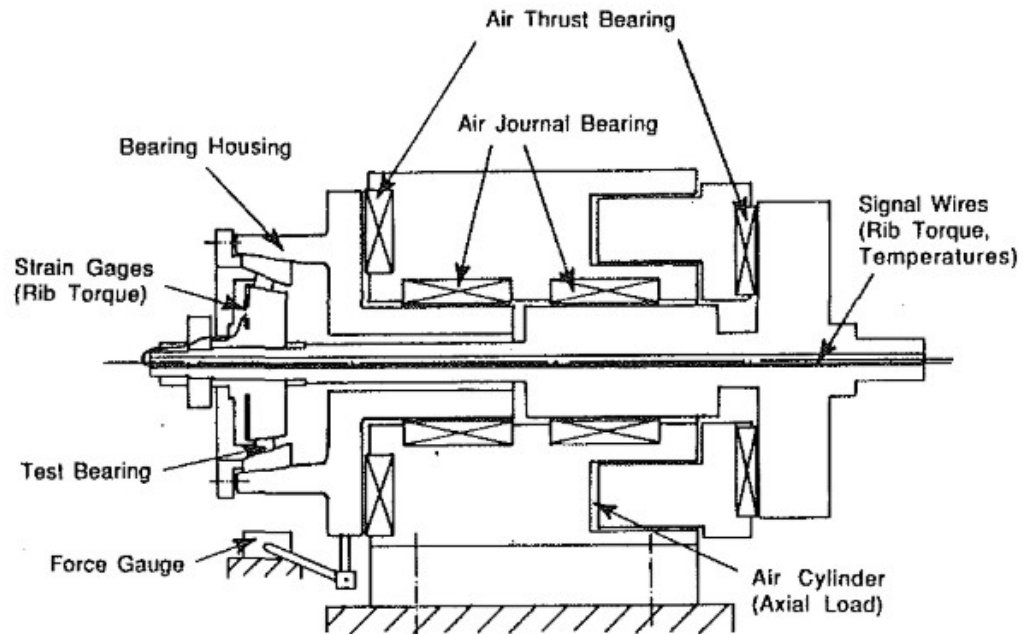


FIGURE 2.12 Bearing Torque Test Rig (Zhou & Hoeprich, 1991)

(Bercea et al., 1996) used an experimental rig, shown in FIGURE 2.13 of his study, to measure the FT of TRBs.

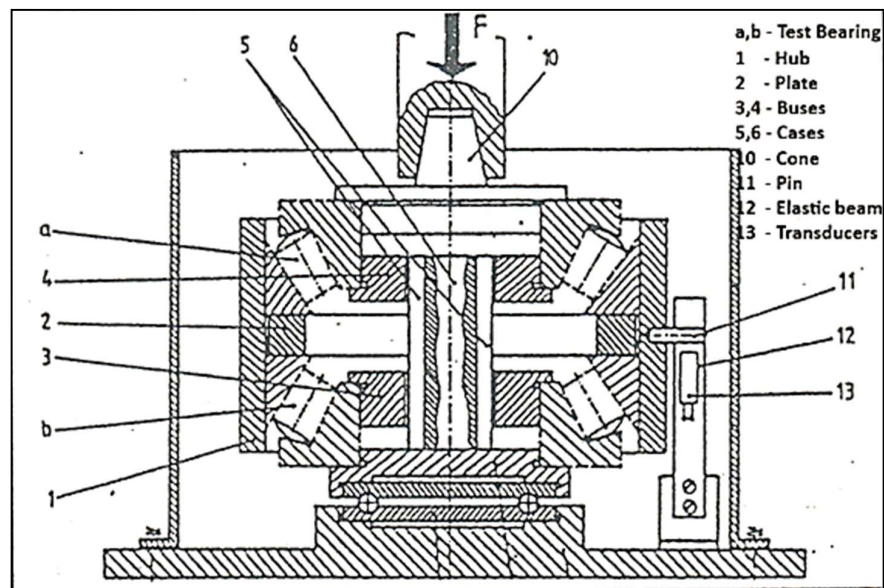


FIGURE 2.13 Measuring Device Through Axial Section (Bercea et al., 1996)

In this arrangement, the first test bearing (a) received rotational motion from the main shaft through the cone, while the second test bearing (b) was driven by bearing (a) through the bushings and the intermediate shaft. Rotation of the casing was restricted by a pin acting on

an elastic beam, which was fitted with two electric resistance transducers mounted on opposite sides. During operation, the pin applied a load to the elastic beam that was proportional to the FT generated on the outer rings of both test bearings. Using this setup, the FT of the 30206 TRB was determined from the torque transmitted to the outer ring, and its variation with both rotational speed and axial load was presented in FIGURE 2.14.

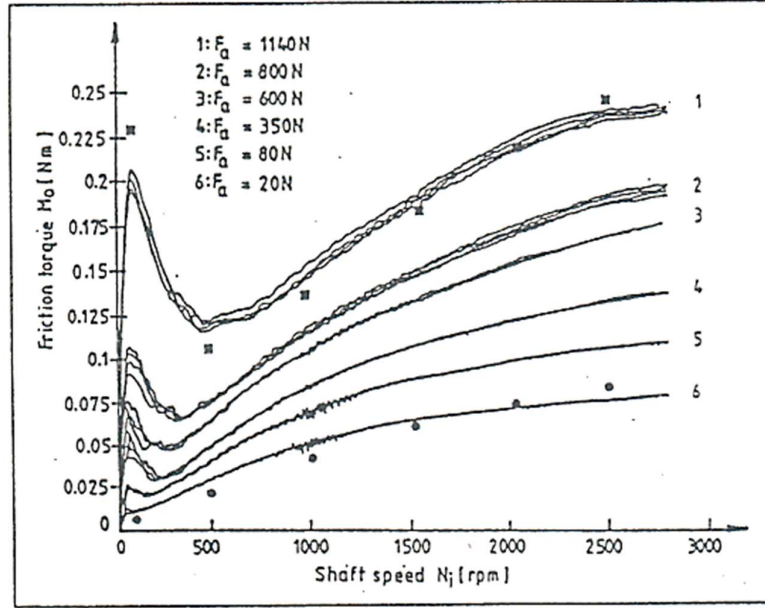


FIGURE 2.14 Predicted FT Measured Values (Bercea et al., 1996)

Matsuyama measured the frictional torque of TRBs in order to verify a formula derived from EHD analysis. The layout of the test rig, presented in FIGURE 2.15 of the study, shows that the rib was separated from the cone, allowing the pure frictional torque generated at the rib contacts to be obtained independently. This rib torque, denoted as m_{rib} , was measured using strain gauges installed as shown in the figure. To determine the total frictional torque of the bearing, m_T , the housing of the test bearing was supported on an air static bearing. This configuration enabled separate measurement of rib-contact torque and total bearing torque under controlled testing conditions (Matsuyama et al., 1998; Matsuyama & Kamamoto, 2001).

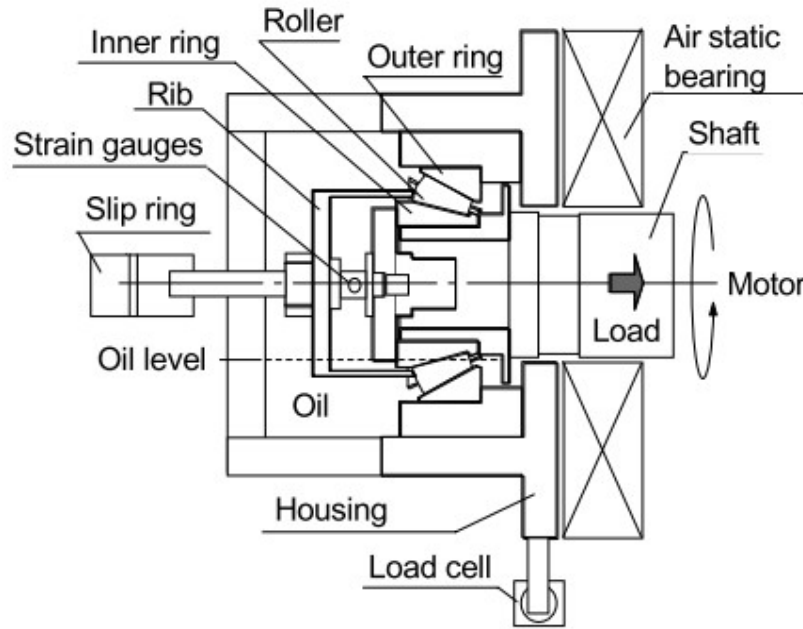


FIGURE 2.15 Bearing Torque Test Rig (Matsuyama & Kamamoto, 2001)

Black proposed a novel experimental arrangement capable of prompt measurement of the running torque of TRBs under combined loading. He presented a test rig design in which a radial load is applied to the system through a central slave bearing, as illustrated in FIGURE 2.16 of his study. In this configuration, springs are used to apply both loads to the shaft, although alternative loading methods can also be used. The radial load is transmitted through two ball bearings positioned midway along the shaft among the two test bearings, ensuring that both test bearings experience the same radial and moment loads. However, the use of slave bearings introduces several fundamental limitations. The friction generated by the slave bearings influences the operating temperature and loading conditions of the test bearings, making it difficult to determine their true thermal behavior. In addition, thermal expansion of the shaft alters the axial load on the fixed-end bearing, while the measured torque includes friction contributions from both the test and slave bearings, reducing the accuracy of FT measurement (Blake & Truman, 2004).

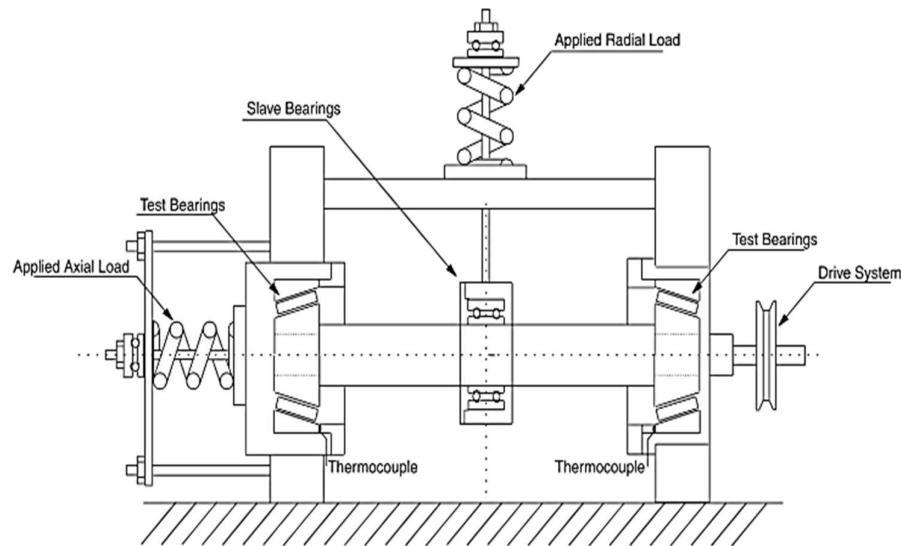


FIGURE 2.16 Two Test Bearing Loading Arrangement (Blake & Truman, 2004)

Cousseau conducted RB tests using a modified Four Ball machine in which the original four ball configuration was replaced by a RB assembly, as illustrated in FIGURE 2.17. The example shown corresponds to a thrust ball bearing, but the revised setup was designed to accommodate various types of RBs lubricated either with oil or grease. This configuration allowed testing of four different bearing types such as thrust ball bearings, TRBs, ACBB, and cylindrical roller thrust bearings. Due to geometric constraints imposed by both the Four-Ball machine and the bearing housing, the maximum allowable bearing dimensions were limited to an outer diameter of 56 mm and a width of 14.3 mm (Cousseau et al., 2010).

M. Mármol Fernández developed a frictional torque test bench capable of realistically reproducing the operating conditions experienced by TRBs, as shown in FIGURE 2.18. The setup measures the frictional torque of the test bearing under combined loads, with the provision to introduce controlled bearing misalignment. The test bearing outer ring is mounted on a hydrostatic bearing, and the frictional torque is determined from the rotational movement of the outer ring using a bending beam force sensor. Radial load is applied through the support bearings of the load module, while axial load is introduced by a hydraulic cylinder.

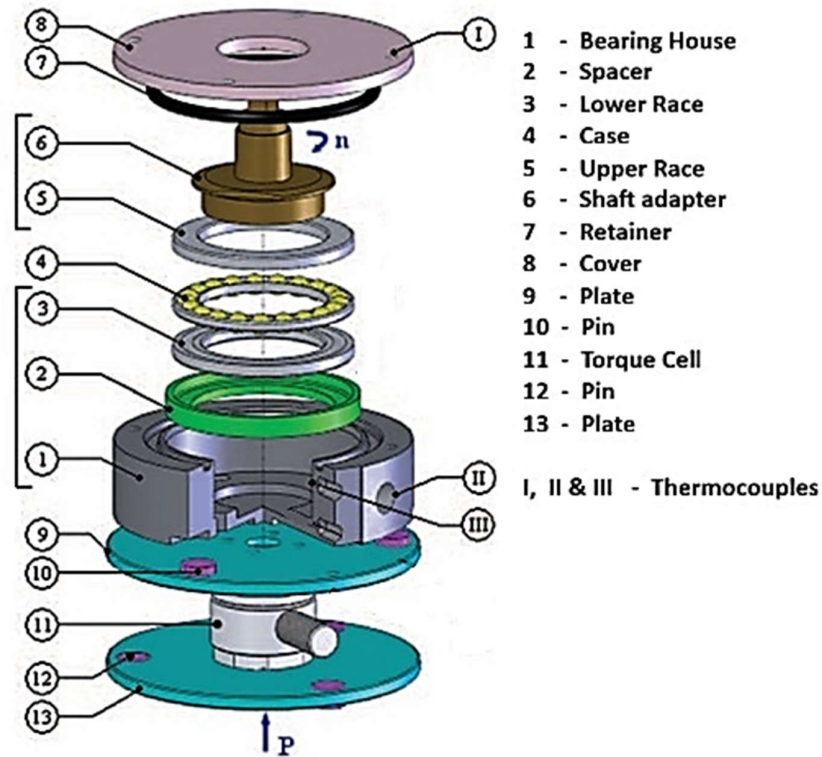


FIGURE 2.17 Thrust Rolling Bearing Assembly (Cousseau et al., 2010)

The test rig also allows independent adjustment of bearing misalignment using a tilt module with linear and rotary guides. The bearing is lubricated using an oil bath system, and the cage speed is measured by inductive sensors mounted on the accessible side of the test bearing. This experimental arrangement enables realistic evaluation of frictional torque under combined loading and misalignment conditions (Mármol Fernández et al., 2021).

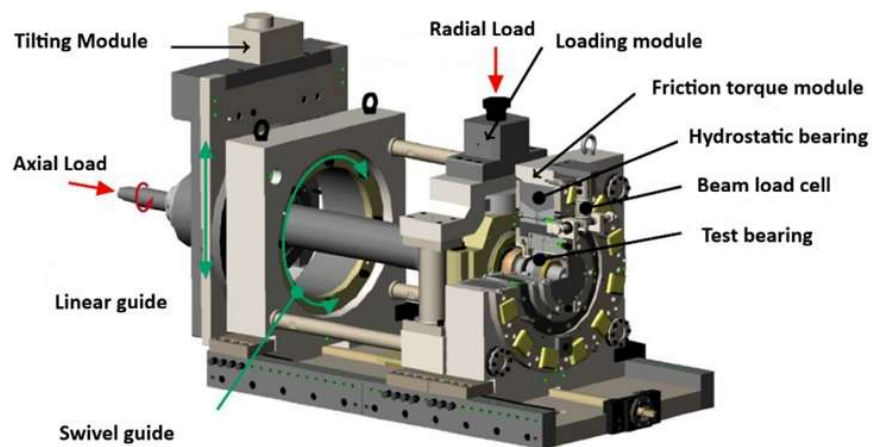


FIGURE 2.18 Frictional Torque Test Bench (Mármol Fernández et al., 2021)

J. Cruz developed an experimental test rig to investigate frictional torque in an automotive axle box, as shown in FIGURE 2.19. The rig consisted of an AC motor with variable speed control, timing pulleys connected by a timing belt, and an upper shaft supported by two ball bearings. The upper shaft was connected to the test axle box through spring couplings and a torque transducer, which continuously measured the transmitted torque and rotational speed. To ensure accurate torque measurements, the complete shaft assembly was precisely aligned using laser alignment technology to minimize misalignment. The axle box was lubricated with SAE 75W90 commercial axle gear oil at the manufacturer-recommended oil level. In addition, a thermos logger equipped with two K-type thermocouples was used to monitor the temperatures of the axle box oil bath and the oil trapped in the pinion shaft bearing chamber throughout the experiments (Cruz et al., 2021).

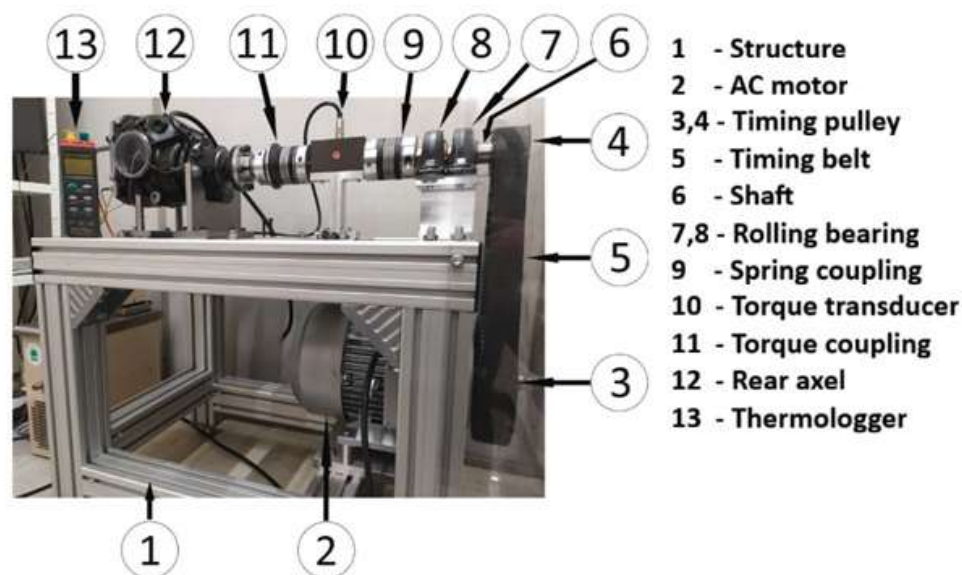


FIGURE 2.19 Torque Test Rig (Cruz et al., 2021)

Manjunath used a RB test machine (RBT), whose cross-section is shown in FIGURE 2.20. The test rig consists of a main shaft driven by an AC motor and supported by one cylindrical roller bearing and two angular contact ball bearings. The test bearing is mounted in a dedicated housing, while axial load is applied using a hydraulic cylinder. FT is measured through a lever arm and load cell connected to the bearing housing, eliminating the influence of the shaft-support bearings. The rig supports both oil-bath and forced lubrication with controlled oil flow, temperature, and level. Thermocouples monitor bearing and lubricant temperatures, and all experimental data are continuously recorded using a digital data

acquisition system. The arrangement is suitable for investigating heavily loaded EHL contacts under pure axial loading (Manjunath et al., 2023).

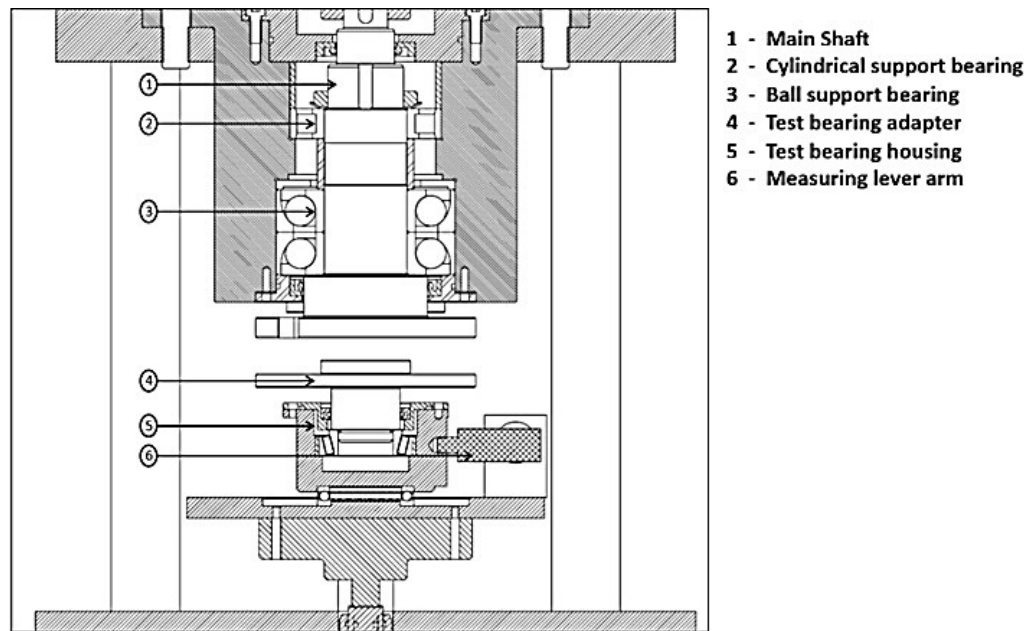


FIGURE 2.20 RB Test Rig Cross Sectional View (Manjunath et al., 2023)

2.8 Conclusion of Literature Review

The key points of the above literature review can be summarized as follows:

- FT is one of the most important performance indicators of TRBs, as it directly influences power loss, heat generation, BT, lubrication performance and service life.
- Researchers have established that FT in TRBs mainly originates from rolling resistance at roller and raceway contacts and sliding friction at rib and roller end contacts, while cage friction and lubricant drag generally contribute less under moderate operating conditions.
- Numerous analytical, numerical, and experimental studies have been developed for FT prediction, considering parameters such as load, speed, lubrication regime, preload, bearing geometry, roller skewing, misalignment, surface roughness, and temperature.
- Most existing FT models are based on EHL theory and contact mechanics, and they provide reasonable predictions under specific operating conditions. However, significant deviations are still reported under high-load, low-speed, and severe lubrication conditions where multiple friction mechanisms interact simultaneously.

- Operating parameters such as rotational speed, load, preload, lubricant viscosity, and temperature have been widely investigated, and their significant influence on FT has been well documented in the literature.
- Several studies have demonstrated that bearing geometry strongly affects FT, including roller profile, raceway geometry, rib design, surface roughness, roller skewing, roundness error, raceway convexity, and geometric homogeneity. However, these investigations are generally limited to individual geometrical parameters rather than their combined effects.
- Recent research trends focus on reducing FT through design modifications, such as optimized rib geometry, surface texturing, micro-dimples, crowning profiles, improved cage designs, and lubrication enhancement techniques. These approaches have shown measurable reductions in friction losses and bearing operating temperatures.

2.9 Research Gaps

Despite extensive use of TRBs in automotive and industrial systems, several gaps remain in the understanding and control of frictional torque, particularly under high axial load conditions:

1. There is a lack of systematic, multivariable experimental investigations that simultaneously consider bearing load, rotational speed, and rib convexity, despite their strong interaction effects on frictional torque and thermal behaviour.
2. Although theoretical and empirical FT models exist, most do not explicitly incorporate rib convexity as a design variable. Consequently, the predictive capability of existing models under varying rib geometries and realistic operating conditions remains limited, particularly for applications requiring accurate torque estimation within controlled temperature limits.
3. While FT is closely linked to heat generation and BT, most optimization efforts focus on torque minimization alone. There is a lack of frameworks that explicitly ensure optimized operating conditions within an acceptable BT range, which is critical for durability and real-world application.

2.10 Research Objectives

Based on the comprehensive review of existing literature and the research gaps identified therein, the primary objectives of the present study are framed as follows:

1. To experimentally evaluate the effect of operating parameters- axial load and rotational speed on frictional torque of TRBs with varying rib convexity.
2. To proposed a predictive and optimization model for frictional torque in TRBs within an acceptable bearing temperature range by systematically evaluating the effects of bearing axial load, rotational speed and rib convexity.
3. To validate the proposed friction torque model through conformation of experiments.

CHAPTER 3

Experimental Setup and Methodology

3.1 Experimental Setup

TRBs are widely used in automotive and industrial applications due to their high load-carrying capacity and ability to sustain combined loads. The performance of TRBs strongly depends on bearing geometry, operating speed, and applied load. In the present study, a specific automotive application of a TRB was considered to determine the bearing size, operating conditions, and corresponding experimental requirements. Based on this application, the bearing specifications were finalized, and a dedicated friction torque (FT) test rig was selected and configured.

The selected single-row TRB has an inner diameter of 90 mm and an outer diameter of 150 mm is assembled inside a vehicle front wheel hub. The maximum loading capacity of the bearing is 6 tons, and the wheel diameter of the considered application is 1008 mm. To ensure accurate, repeatable, and reliable experimental outcomes, the test rig was carefully assembled, calibrated, and validated before conducting the experiments.

3.2 Friction Torque (FT) Test Rig Description

The assembly of the FT test rig used for the experimental investigation is illustrated in FIGURE 3.1, while the overall control architecture based on the Supervisory Control and Data Acquisition (SCADA) system. The test rig is specifically designed to measure the FT characteristics of TRBs under controlled axial loading and operating speed conditions.

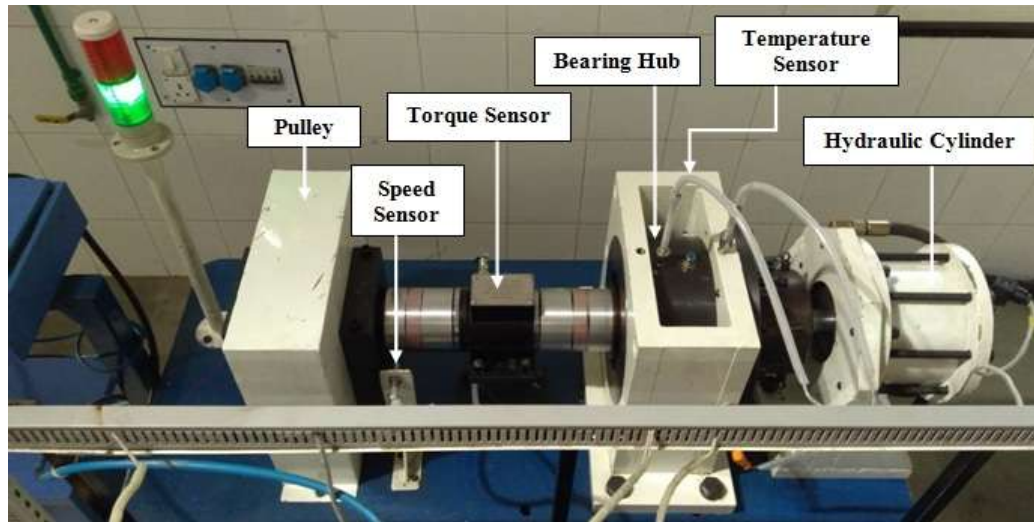


FIGURE 3.1 Friction Torque Test Rig

The experimental setup developed for the present investigation consists of five major units, namely the drive unit, test bearing unit, coupling and torque measurement unit, hydraulic load unit, and control/data acquisition unit. The complete arrangement was designed to evaluate the tribological and thermal performance of the TRBs under varying operating conditions of load and rotational speed. The test rig enables precise control of experimental parameters and accurate measurement of FT, BT, axial load, and rotational speed during experimentation.

The main components of the test rig are as follows:

3.2.1 Drive Unit:

The drive unit is the primary power transmission section of the experimental test rig, providing controlled rotational motion to the test bearing assembly. It consists of an AC induction motor coupled with a pulley and belt transmission system, enabling efficient speed transmission and vibration reduction during operation. The motor speed is regulated through a Variable Frequency Drive (VFD), allowing precise adjustment of rotational speed according to experimental requirements. The motor shaft is supported by two deep groove ball bearings that ensure smooth rotation, proper alignment, and minimal frictional losses. The output shaft is connected to the test bearing shaft through a coupling arrangement, supplying continuous and stable rotational motion to simulate actual operating conditions encountered in industrial and automotive applications.

3.2.2 Test Bearing Unit (Bearing Hub):

The test bearing unit is the core section of the experimental setup, comprising the TRBs under investigation, a support bearing, a bearing holder, and a specially designed housing. The test bearing is securely mounted with back to back arrangement with the other supported bearing within the housing using a precision-machined holder to ensure accurate alignment, proper load distribution, and stable operation during testing. An adjacent support bearing provides additional shaft stability, minimizes shaft deflection, and maintains alignment during high-speed operation. The rigid housing structure withstands axial and rotational loads while ensuring dimensional stability under varying operating conditions. The unit is designed for easy bearing installation and replacement, facilitating multiple experimental trials and enabling accurate transmission of axial load from the hydraulic loading system to the test bearing.

3.2.3 Coupling Unit and Torque Measurement:

The drive unit and test bearing unit are connected through a coupling assembly consisting of two backlash-free flexible couplings and a high-precision torque sensor. The flexible couplings compensate for minor shaft misalignments while ensuring smooth, vibration-free transmission of torque and minimizing additional stresses on the shaft and bearing assembly. Positioned between the couplings, the torque sensor continuously measures the FT generated within the test bearing under varying load and speed conditions. The measured data are transmitted to the data acquisition system for real-time monitoring and analysis. Accurate torque measurement is essential for evaluating bearing performance, as FT directly affects power losses, lubrication effectiveness, and heat generation. To ensure reliable results, the torque sensor was calibrated before experimentation and the entire assembly was carefully aligned to minimize measurement errors.

3.2.4 Hydraulic Load Unit:

The hydraulic load unit is designed to apply controlled axial loads to the TRB during experimentation. It consists of a hydraulic cylinder, hydraulic pump, flow control valve, load cell, and thrust support arrangement. The hydraulic cylinder applies the required axial force to the bearing assembly, while the flow control valve enables precise adjustment of the load for different testing conditions. This system ensures smooth and uniform load application without sudden shocks, thereby improving the stability and reliability of experimental

results. A load cell continuously monitors the applied axial load and transmits the data to the SCADA system for real-time recording and visualization. To safely withstand the externally applied force, a thrust ball bearing with higher axial load-carrying capacity than the test bearing is incorporated into the assembly, ensuring smooth load transfer and protecting system components from excessive stress. The hydraulic loading system effectively simulates the operating conditions encountered by TRBs in industrial and automotive applications.

3.2.5 Control and Data Acquisition Unit:

The experimental setup is integrated with a SCADA based control and monitoring system as shown in FIGURE 3.2. This unit is responsible for real-time measurement, visualization, recording, and storage of experimental parameters throughout the testing process.

The SCADA system continuously monitors important operating parameters including:

- Friction Torque (FT)
- Bearing Temperature (BT)
- Axial Load
- Rotational Speed (rpm)

Different sensors are integrated with the setup for parameter measurement. A torque sensor is used for FT measurement, thermocouple or temperature sensor is directly attached to the outer diameter (OD) surface of the outer ring of the test TRB for continuous monitoring of the bearing temperature during experiments, a load cell measures axial load, and a speed sensor or proximity sensor records shaft rotational speed. The acquired sensor signals are processed through the data acquisition hardware and displayed on the SCADA interface in real time. The system also records the data for further analysis and comparison. Continuous monitoring helps in identifying performance trends, thermal behaviour, lubrication effectiveness, and operating stability of the bearing under varying conditions. Prior to experimentation, all measuring instruments and sensors were calibrated using standard calibration procedures to ensure high accuracy, precision, and reliability of data. Calibration minimizes systematic errors and enhances confidence in the experimental results. The SCADA-based system significantly improves experimental efficiency by enabling automated data logging, reducing human error, and facilitating accurate performance analysis of the TRB under different operating conditions.

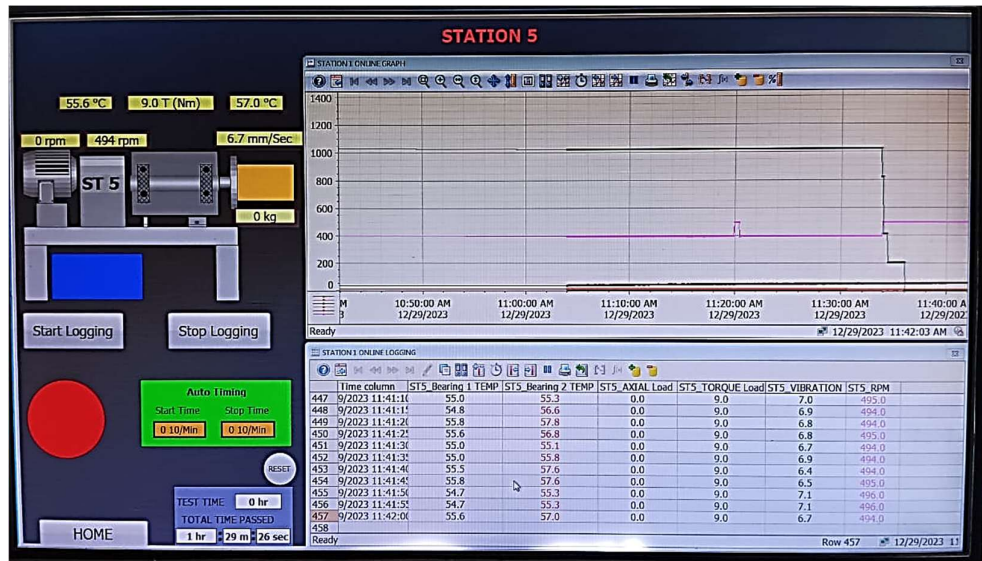


FIGURE 3.2 The SCADA System

3.3 Test Rig Features and Technical Specifications

The developed experimental test rig was designed to evaluate the performance characteristics of TRBs under controlled operating conditions of axial load and rotational speed. The setup is capable of accommodating TRBs ranging up to 135 mm bore diameter and 200 mm outer diameter. The technical specifications of the experimental test rig were selected based on the dimensional and operational requirements of the selected TRB, as well as the intended range of experimental conditions. However, the bearing inner diameter of 90 mm and outer diameter of 150 mm were selected according to the dimensions of the test TRB (33118) and accordingly, a customized bearing hub assembly was designed and fabricated to accommodate a test TRB.

The test rig is equipped with a hydraulic loading system capable of applying axial loads up to 10,000 kgf. This high-capacity loading arrangement enables the simulation of severe operating conditions encountered in heavy-duty industrial and automotive applications. The hydraulic system provides smooth and controlled load application, ensuring stable testing conditions and reliable experimental measurements.

The rotational motion of the shaft is provided through an AC motor drive system, and the shaft speed can be continuously varied up to a maximum of 1200 rpm using a variable

frequency drive (VFD). The ability to regulate speed allows investigation of bearing behaviour under different operating regimes and lubrication conditions.

FT generated within the test bearing is measured using a high-precision torque sensor having a maximum measurement capacity of 500 Nm. The torque measurement system enables accurate evaluation of frictional behaviour, power losses, and lubrication performance during experimentation.

The lubrication system employed in the present work uses grease lubrication. High Lub LT 2 EP grease was selected as the lubricant for the test TRB, with an approximate initial grease quantity of 50 g, based on the recommended lubrication requirements for the bearings. The grease was selected considering its suitable extreme-pressure characteristics, load-carrying capacity, thermal stability, and compatibility with TRB applications. Proper lubrication ensures reduced wear, minimized friction, and stable thermal performance during testing.

The major technical specifications and operating capabilities of the developed test rig are listed in TABLE 3.1.

TABLE 3.1 Technical Specifications of Test Rig

Parameters	Values
Bearing ID (mm)	Up to 135 mm
Bearing OD (mm)	Up to 200 mm
Axial Load (kgf)	10,000 Max
Bearing Speed (rpm)	1200 Max
Running Torque (Nm)	500 Max
Lubrication	High Lub LT 2 EP Grease

The developed test rig provides a reliable and flexible platform for investigating the tribological and thermal performance of TRBs under varying operating conditions. The robust construction, precise loading mechanism, and SCADA-based monitoring system ensure high experimental accuracy, repeatability, and operational safety during experimentation.

3.4 Selection of TRB for the Experimental Study

The geometric configuration of the single-row TRBs employed in the present study is illustrated in FIGURE 3.3. Among the commercially available TRBs of different sizes and load capacities, the single-row TRB (33118) was selected for the present study based on the compatibility with the test rig, suitability for precision modification, ability to generate measurable frictional behavior, and industrial relevance. The TRB (33118) bearing was also selected as it is a medium-sized industrial TRBs capable of carrying high axial load while generating measurable FT under different operating conditions. Compared with smaller bearings, its relatively larger rib and roller contact area produces sufficient heat generation and frictional effects, enabling accurate measurement of the influence of inner ring rib geometry. At the same time, it is considerably more economical and easier to manufacture than larger heavy-duty TRBs. Furthermore, the selected bearing is widely used in automotive, industrial gearbox, and heavy machinery applications where high axial loads and rotational speeds are commonly encountered. Consequently, the experimental results obtained from this bearing are representative of practical engineering applications and can be extended to similar classes of TRBs.

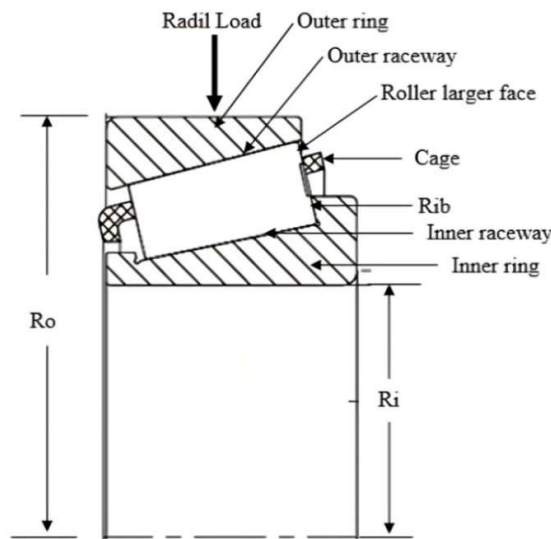


FIGURE 3.3 Tapered Roller Bearing

The corresponding cross-sectional and dimensional view of the bearing is shown in FIGURE 3.4 and the dimensional specifications and major geometrical parameters of the selected TRB is presented in TABLE 3.2.

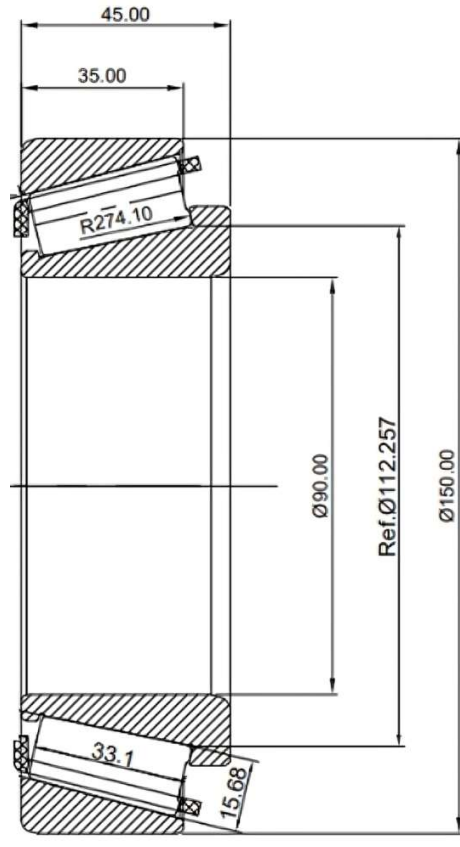


FIGURE 3.4 TRB Cross Sectional View

TABLE 3.2 TRB (33118) Bearing Specification

Parameters	Values
outer diameter (nominal)	150 mm
inner diameter (nominal)	90 mm
inner ring width (nominal)	45 mm
outer ring width (nominal)	35 mm
Total number of rollers	23
Bearing material	SAE52100

3.5 TRB (33118) Bearing Parametric Specifications

For all tested bearings as shown in TABLE 3.3, the convexity of the outer raceway, inner raceway, and roller surfaces was carefully controlled and maintained at maximum values of 6 μm , 6 μm , and 4 μm , respectively. Maintaining controlled convexity helps in improving contact stress distribution between mating surfaces and minimizing localized stress concentration during operation. In addition to geometric control, surface finish quality was also maintained uniformly across all bearing components. The surface roughness (Ra) of the raceways, roller surfaces, and rib regions was controlled to a maximum value of 0.2 μm . The low surface roughness ensured stable lubrication film formation, minimized frictional variations, and provided consistent tribological conditions throughout experimentation.

TABLE 3.3 TRB (33118) Bearing Parametric Specification

Parameters	Values
Outer raceway convexity	6 μm max
Outer raceway surface roughness (Ra)	0.2 μm max
Inner raceway convexity	6 μm max
Inner raceway surface roughness (Ra)	0.2 μm max
Roller Length	33.10 mm
Roller convexity	6 μm max
Roller surface roughness (Ra)	0.2 μm max
Inner Race Rib Ra	0.2 μm
Inner Race Rib Convexity	vary

For this purpose, multiple inner raceways having different rib convexity values were manufactured through precision machining and finishing operations. The rib profiles were then measured and verified using a TALYSURF surface roughness and profile measurement system. The TALYSURF instrument enabled accurate measurement of micro-geometric variations and ensured precise evaluation of rib convexity profiles.

3.6 Validation of Test Rig

Before conducting the detailed experimental investigation, the developed TRB test rig was validated to ensure the reliability, accuracy, and stability of the experimental setup. The validation process was carried out to confirm that the test rig was capable of producing consistent thermal and tribological behaviour comparable with standard bearing operating characteristics reported in bearing catalogues.

3.6.1 Measurement Accuracy and Uncertainty of Experimental Instruments

The accuracy of the major measuring instruments used in the experimental test rig was assessed from their calibration certificates. The dynamic torque sensor, hydraulic pressure gauge, non-contact RPM meter and thermocouple were calibrated. The Calibration reports are included in Appendix B. The maximum observed errors / uncertainty of experimental instruments were ± 0.4 Nm for the torque sensor, ± 0.4 kg/cm² for the hydraulic pressure gauge, ± 5 rpm for the RPM meter and 0.843° C for thermocouple. Corresponding to maximum percentage errors of 0.8%, 0.2%, 0.3%, and 0.84 respectively. All four instruments were reported as acceptable according to the specified calibration acceptance criteria.

3.6.2 Thermal behaviour comparison with standard TRBs operating characteristics

For the further validation, a single row TRB (33118) was selected and mounted on the test rig under controlled operating conditions. The bearing was operated continuously for approximately 20 hours while monitoring the variation of BT with operating time through the SCADA based data acquisition system. During the validation test, BT readings were recorded continuously at regular intervals. The obtained temperature trend from the developed test rig was then compared with the standard operating temperature trend available in the SKF bearing catalogue (SKF Groups, 2018). The comparison was primarily based on the relationship between operating time and BT rise under continuous running conditions.

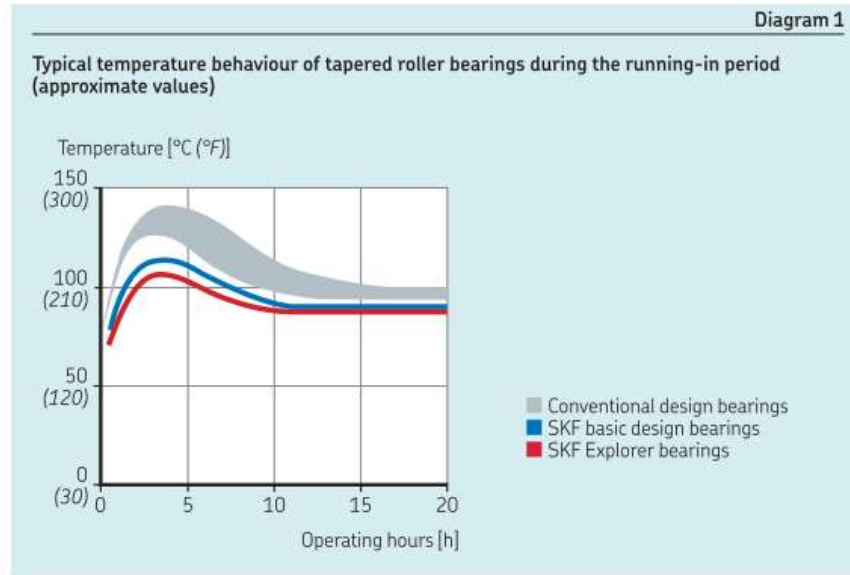


FIGURE 3.5 SKF TRB Temperature Behaviour During Running Period (SKF Groups, 2018)

The standard SKF catalogue graph showing the variation of operating hours versus BT is presented in FIGURE 3.5, while the corresponding graph obtained from the developed experimental test rig is shown in FIGURE 3.6.

It was observed that the temperature variation pattern obtained from the developed test rig closely matched the standard SKF temperature trend. In both cases, the BT increased gradually during the initial running period due to frictional heat generation and subsequently approached a relatively stable operating temperature after continuous operation.

The similarity between the SKF standard graph and the experimentally obtained graph confirmed that the thermal behaviour of the bearing on the developed test rig was consistent with standard bearing operating characteristics. This agreement validated the capability of the developed experimental setup to accurately simulate actual operating conditions of TRBs.

Furthermore, the validation trial confirmed the proper functioning of the drive system, hydraulic loading arrangement, temperature sensing system, and SCADA-based monitoring unit. No abnormal vibration, excessive noise, or instability was observed during continuous operation of the test rig.

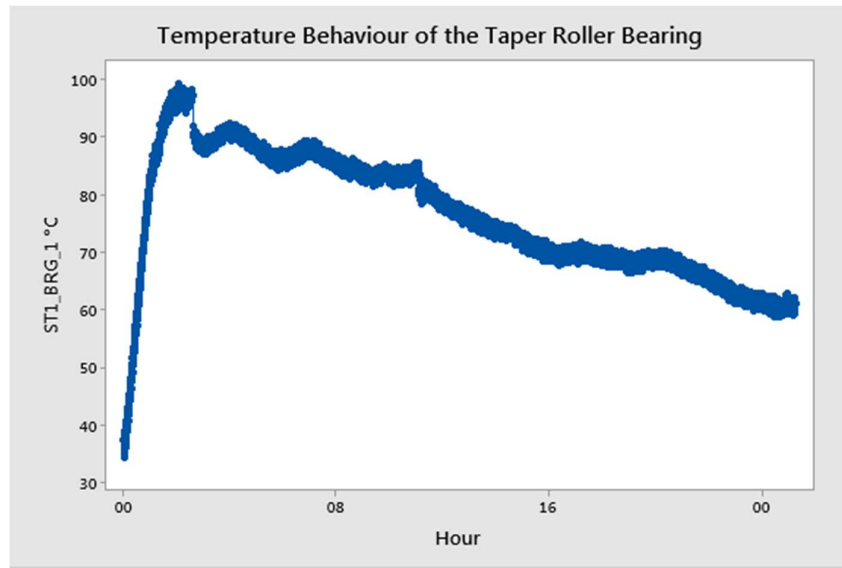


FIGURE 3.6 Test Rig Tested TRB 33118 Temperature Behaviour During Running Period

The successful validation of the test rig established confidence in the reliability and repeatability of the experimental setup for conducting the detailed investigation on the influence of rib convexity, axial load, and operating speed on TRB performance.

3.7 Research Considerations and Assumptions

To ensure reliability, repeatability, and consistency of the experimental investigation, several research considerations and assumptions are taken into account for the design and execution of the experiments. These assumptions helped in simplifying the experimental analysis while maintaining realistic operating conditions for the TRBs.

- **Material Properties:** The materials used for all the components of the bearing and test rig are homogeneous and isotropic.
- **Temperature Stability:** The experiments are conducted in a controlled environment where temperature fluctuations are minimal, ensuring that thermal expansion or contraction does not affect the results.
- **Load Application:** The loads applied during the experiments are static and they are applied uniformly and symmetrically to avoid uneven stress distribution.
- **Bearing Alignment:** The test bearing is perfectly aligned with the test rig to prevent additional stresses or misalignment errors.

- Negligible Wear and Tear: The bearings and rig components are assumed to be free from wear and tear at the start of the experiment, and any changes due to wear during the experiment are negligible.
- Friction and Lubrication: The effects of friction and lubrication are assumed to be constant throughout the experiment. The lubrication is consistent and adequate.
- Vibration and Noise: External vibrations and noise are minimized, and any vibrations in the system are intrinsic and measured.
- Measurement Accuracy: The instruments used for measuring load, torque, speed and temperature are properly calibrated and have negligible error margins.
- Repeatability: The test setup allows for repeatable experiments, ensuring that any variations in results are due to the experimental conditions and not due to inconsistencies in the setup.

3.8 Conclusion

This chapter presented the experimental methodology developed for investigating FT and BT of TRB 33118. The customized test rig incorporates a drive unit, bearing hub, hydraulic loading system, flexible coupling, torque sensor, and SCADA based data acquisition system for continuous measurement of FT, BT, axial load, and rotational speed. The test rig was validated through measurement accuracy and uncertainty of experimental instruments and comparison of the measured temperature behaviour with the standard SKF bearing temperature profile. For the performing and ensuring reliability, repeatability, and consistency the experiments, several research considerations and assumptions are taken in to account.

CHAPTER 4

Design of Experiments

4.1 Design of Experiments (DOE)

Mechanical experimentation generally involves significant physical effort, material consumption, machine running costs, and time. If experiments are not planned systematically, a large number of trials may be required, resulting in increased cost and resource utilization. DOE is a systematic approach utilized to design and perform experiments efficiently and enables the study of several factors simultaneously with a minimum number of experiments, thereby reducing experimental effort, time, and overall cost while ensuring reliable and meaningful results and achieving the desired objective, such as optimization. It helps in selecting the most suitable experimental conditions while reducing numbers of experiments required. Since experimental results are often influenced by unavoidable variations (noise), statistical methods are used to analyze the data and obtain reliable outcomes (Fisher, 1974; Montgomery, 2013).

The three basic principles of DOE are replication, randomization and blocking. Replication improves the accuracy of the results and helps evaluate experimental error. Randomization reduces the effect of uncontrolled factors by performing experiments in a random order. Blocking makes groups of similar experimental conditions together to reduce the influence of known sources of variation and improve the precision of the results (Cavazzuti, 2013).

DOE is closely associated with statistical analysis and has become an important tool to optimize the process and improve the quality. It was first developed by Sir Ronald Aylmer Fisher in the 1920s and was later advanced by Box and Wilson in 1951 through the introduction of Response Surface Methodology (RSM). In the 1980s, Genichi Taguchi further promoted the application of statistical experimental design to improve product and process quality (Anderson & McLean, 2018; Mead, 1988).

Several DOE techniques such as Full Factorial Design (FFD), Fractional Factorial Design, Taguchi Design, Randomized Complete Block Design, Latin Square design, RSM etc.

Factorial Design and RSM are commonly employed by researchers for planning and analysing experimental investigations (Ilzarbe et al., 2008; Kleijnen, 2008).

The performance of TRBs is influenced by several operating and geometrical parameters that may interact with each other. The present investigation involves multiple controllable factors rib convexity, axial load, and rotational speed and their effects on two responses, namely FT and BT. DOE provide a systematic and statistically based approach for studying the individual and combined effects of these factors while reducing the number of experimental trials compared with conventional methods one factor at a time experimentation. It also enables identification of significant factors and interactions, development of predictive mathematical models, and subsequent optimization of the process parameters. Therefore, DOE was considered more suitable than conventional approaches for achieving the objectives of the present experimental investigation. The DOE based FFD and RSM methodology adopted in this work provides a systematic and effective framework for evaluating and optimizing the performance of selected test TRB.

The primary response parameters considered in this study are FT and BT. To investigate the individual and interaction effects of the selected factors, a FFD was initially employed. Subsequently, RSM used to develop predictive models and determine the optimum operating conditions.

4.2 Pre Experimental Planning

Pre-experimental planning is a crucial stage in the design of experiments, as it establishes the framework for conducting systematic and reliable investigations. Proper planning ensures that the experimental objectives are clearly defined, the variables influencing the system are appropriately identified, and the experiments are designed to generate meaningful and statistically valid results. Careful selection of the response variable and operating parameters not only improves the efficiency of experimentation but also minimizes unnecessary trials, experimental errors, and resource consumption (Farooq et al., 2016; Montgomery, 2013).

4.2.1 Pilot Experiments and Standard Operating Procedure

Based on the literature review, bearing operating characteristics, and the objectives of the present investigation, rib convexity, axial load, and operating speed were identified as the primary factors influencing the thermal and tribological performance of TRBs. Before finalizing the levels of these parameters for the detailed experimental study, pilot experiments were conducted to establish safe operating ranges and evaluate their influence on bearing FT and temperature.

The pilot experiments were performed using a single-row TRB (33118) mounted in a back-to-back arrangement with a spacer between the inner faces of the bearings. A preload of 120 N was applied through the lock nut arrangement to eliminate internal clearance, while the end play was maintained at zero. The initial shaft running torque after preload was approximately 5 Nm.

A standard operating procedure (SOP) was followed throughout the experimentation. After mounting the bearing and installing the sensors, the alignment of the shaft, coupling assembly, hydraulic loading system, and measuring instruments was verified. The machine was then operated at the desired speed, and the required axial load was applied through the hydraulic loading system while continuously monitoring the operating parameters through the SCADA interface.

FT, BT, axial load, and rotational speed were continuously recorded during testing. Particular attention was given to abnormal operating conditions. If a sudden increase in BT and/or FT was observed, the test was discontinued and the bearing assembly was inspected. The limiting temperature of 120°C was selected based on the permissible operating temperature range recommended in the SKF Catalogue (2018) [58].

During the pilot investigation as shown in TABLE 4.1, The experiments were conducted over a wide range of operating conditions as the axial load was varied from 1000 to 8000 kgf. The axial load values in the present experimental investigation are obtained directly from the hydraulic load unit/test rig, which displays and records the applied load in kgf. Therefore, kgf has been retained throughout the experimental data, tables, graphs, and statistical analysis to maintain consistency with the actual measured data and analysis. The corresponding SI force conversion $1 \text{ kgf} = 9.80665 \text{ N}$ has been provided where appropriate

for clarity. And the bearing speed was varied from 100 rpm to 800 rpm to determine the safe and practical limits of axial load and rotational speed for the subsequent DOE.

TABLE 4.1 Pilot Experiments Investigation Data

Load (kgf)	Speed (rpm)	Torque Max (Nm)	Torque Min (Nm)	Tem Min (°C)	Tem Max (°C)	Time (min)
1000	400	21	11	40	91	120
2000	400	23	16	38	50	120
4000	400	24	16	38	57	120
6000	400	25	17	38	66	120
2000	600	26	14	51	80	90
4000	600	28	16	44	79	90
6000	600	32	22	45	96	90
2000	800	29	16	65	90	60
4000	800	31	24	53	90	51
6000	800	35	32	52	90	24
7000	400	31	25	54	106	70
8000	400	35	31	57	108	58

The rib convexity was maintained at 0 μm for these preliminary trials. Different combinations of load and speed were tested to evaluate their effects on FT, BT, and overall operating stability. The observations indicate that FT and BT increased consistently with increasing axial load and rotational speed, while the permissible test duration decreased significantly.

At a constant speed of 400 rpm, increasing the axial load from 2000 kgf to 6000 kgf resulted in only a moderate increase in maximum FT (23–25 Nm) and BT (50–66°C), while stable operation was maintained for 120 minutes. However, at higher loads of 7000 kgf and 8000 kgf, the maximum BT increased rapidly to 106°C and 108°C, respectively, with FT exceeding 30 Nm and the allowable running time reducing to 70 and 58 minutes. Post-test

examination revealed friction marks and surface damage on the raceway and rib contact regions, indicating excessive contact stress and heat generation.

Similarly, increasing the speed from 400 rpm to 800 rpm caused a substantial rise in both FT and BT. At 6000 kgf and 800 rpm, the maximum FT reached 35 Nm and the test duration reduced drastically to only 24 minutes, indicating unstable operating conditions. Since FT values above 20 Nm are considered relatively high and BTs rapidly approaching or exceeding 100°C is undesirable for reliable bearing operation, higher load-speed combinations were considered unsuitable for further experimentation.

Based on these observations, the maximum factor levels were restricted to 6000 kgf axial load and 600 rpm rotational speed for the detailed DOE. These operating limits provided stable test conditions while avoiding excessive FT, excessive temperature rise, and premature termination of the experiments, thereby ensuring safe operation and reliable experimental data.

4.2.2 Selection of Factors and Levels

The selection of experimental factors and their corresponding levels is a critical step in the DOE methodology. The factors selected for the present investigation were finalized based on an extensive literature review, bearing operating requirements, test rig limitations, and observations obtained from the pilot experiments conducted on the developed TRB test rig.

During the pilot experimentation, various combinations of axial load, rotational speed, and rib convexity were investigated to identify their influence on bearing FT, operating temperature, and contact stress. The pilot study revealed that these parameters significantly affect the FT and BT of TRBs. The experimental observations indicated that BT and FT increased with increasing axial load and rotational speed. It was also observed that operating conditions involving axial loads greater than 6000 kgf in combination with rotational speeds above 600 rpm resulted in excessive heat generation and rapid temperature rise. Furthermore, contact stress analysis showed that rib convexity values greater than 6 μm produced contact stresses beyond the acceptable limits recommended by bearing standards. Based on these observations, three process parameters were selected as the input factors for the present study:

A. Rib Convexity

The rib convexity of the inner raceway was selected as the primary geometric parameter because it directly influences the contact conditions between the roller end and the rib surface. Variation in rib convexity affects contact stress distribution, lubrication characteristics, friction generation, and heat dissipation within the bearing. Three rib convexity levels were selected based on contact stress analysis and pilot study observations.

B. Axial Load

Axial load was selected as the second experimental factor because TRBs are predominantly designed to carry axial loads in practical applications. The applied load significantly influences contact pressure, frictional resistance, BT, and results affects to the bearing life. The selected load levels were chosen within the safe operating range established during pilot testing.

C. Operating Speed

Rotational speed was selected as the third experimental factor because it has a direct influence on lubricant behaviour, FT generation, heat generation, and overall bearing performance. The selected speed levels represent low, medium, and high operating conditions within the validated operating range of the test rig. Based on the pilot experimental observations and operational constraints, each factor was considered at three levels for the detailed experimental investigation. The selected factors and their levels are presented in TABLE 4.2.

TABLE 4.2 Factors and Levels for Final Experiments

Factors	Level			Code
	1	2	3	
Rib Convexity (μ)	0	3	6	A
Axial Load (kgf)	2000	4000	6000	B
Bearing Speed (rpm)	400	500	600	C

The selected factor levels ensure safe operation of the bearing system while providing sufficient variation to evaluate the individual and interactive effects of rib convexity, axial load, and operating speed on FT, BT, and contact stress. These factors and levels were

subsequently utilized for developing the experimental design matrix and conducting the detailed investigation.

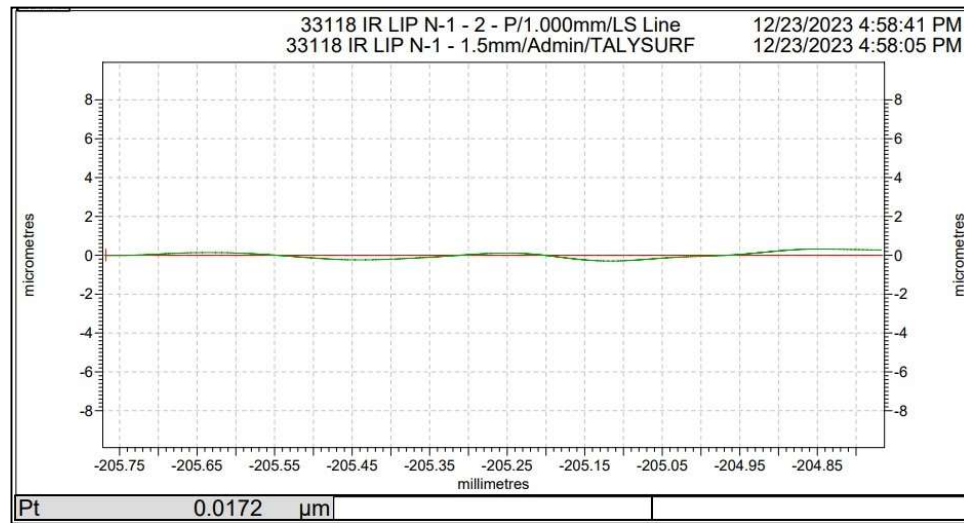


FIGURE 4.1 Rib Convexity Profile 0 μm (Approx)

A number of inner raceways were manufactured with needed rib convexities and measured with TALYSURF profilometer. Among these all three inner raceways having rib convexity values of approximately 0 μm , 3 μm , and 6 μm as shown in FIGURE 4.1, FIGURE 4.2 and FIGURE 4.3 respectively selected were assembled separately with identical bearing components to prepare three test bearings for final experimental evaluation. The controlled variation in rib convexity enabled systematic investigation of its influence on FT and BT under different operating conditions.

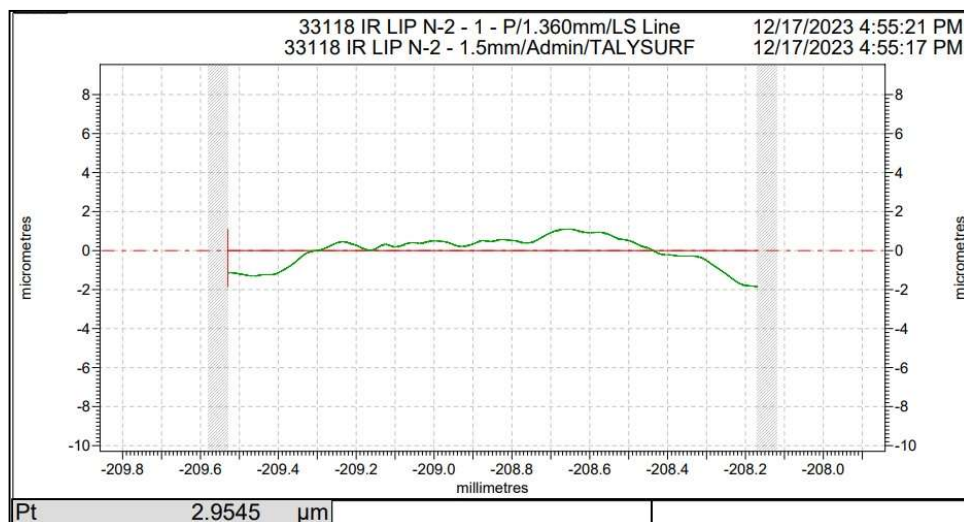


FIGURE 4.2 Rib Convexity Profile 3 μm (Approx)

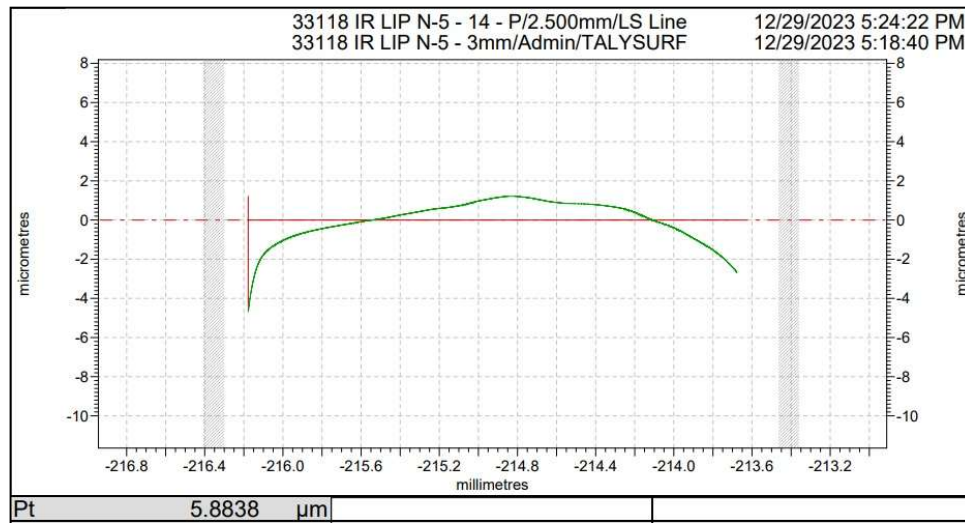


FIGURE 4.3 Rib Convexity Profile 6 μm (Approx)

The careful preparation, dimensional verification, and surface inspection of the test bearings ensured high accuracy, repeatability, and consistency in experimental observations, thereby improving the reliability of the obtained results and subsequent analysis.

4.3 Full Factorial Design (FFD) using MINITAB 19

The FFD is one of the most widely used approaches because it allows the evaluation of all possible combinations of factor levels and provides comprehensive information regarding both the individual and interaction effects of the selected factors (Montgomery, 2013). Minitab is a statistical software and powerful statistical analysis tool widely used for quality improvement, process optimization, and data analysis. It provides a user-friendly platform for applying statistical techniques, identifying trends, analyzing experimental data, and supporting informed decision-making in research and industrial applications (Mathews, 2004; Montgomery, 2010).

In the present study, a FFD was adopted to investigate the influence of rib convexity, axial load, and operating speed on the performance of TRBs using MINITAB 19 software. The selected response parameters were FT in Nm and BT in $^{\circ}\text{C}$. Since each of the three factors was considered at three levels, a (3^3) FFD consisting of 27 experimental combinations was generated. Thus, an L27 FFD was employed for conducting the experiments.

TABLE 4.3 shows the L27 FFD with 3 levels and 3 factors. Here Std. Order shows the standard sequence of the experimental combinations generated by the DOE while Run Order shows the randomized sequence in which the experiments were actually performed to minimize the effect of experimental bias arising from uncontrolled variables and to ensure mutual exclusivity among the trials.

TABLE 4.3 L27 FFD with 3 Level & 3 Factors

Std. Order	Run Order	A	B	C
1	26	1	1	1
2	25	1	1	2
3	8	1	1	3
4	14	1	2	1
5	6	1	2	2
6	20	1	2	3
7	23	1	3	1
8	18	1	3	2
9	1	1	3	3
10	19	2	1	1
11	24	2	1	2
12	12	2	1	3
13	22	2	2	1
14	3	2	2	2
15	13	2	2	3
16	7	2	3	1
17	16	2	3	2
18	21	2	3	3
19	5	3	1	1
20	2	3	1	2
21	4	3	1	3
22	11	3	2	1
23	10	3	2	2
24	9	3	2	3
25	27	3	3	1
26	15	3	3	2
27	17	3	3	3

4.4 Experimental Results and Analysis

To improve the reliability and repeatability of the experimental results obtained from L27 FFD, Three repeated trials were conducted for each experiment as shown in table presented in Appendix C. In this table the individual observations obtained from the three trials were recorded and the mean and standard deviation (SD) were calculated for FT and BT.

TABLE 4.4 L27 Full Factorial DOE with Experimental Results FT and BT

Std. Order	Run Order	A	B	C	FT	BT
1	26	1	1	1	12	65
2	25	1	1	2	14	72
3	8	1	1	3	15	79
4	14	1	2	1	13	74
5	6	1	2	2	15	80
6	20	1	2	3	16	87
7	23	1	3	1	15	79
8	18	1	3	2	17	86
9	1	1	3	3	19	96
10	19	2	1	1	8	55
11	24	2	1	2	10	61
12	12	2	1	3	11	65
13	22	2	2	1	9	61
14	3	2	2	2	12	67
15	13	2	2	3	14	73
16	7	2	3	1	10	64
17	16	2	3	2	13	72
18	21	2	3	3	15	79
19	5	3	1	1	7	67
20	2	3	1	2	10	73
21	4	3	1	3	11	81
22	11	3	2	1	8	74
23	10	3	2	2	11	79
24	9	3	2	3	13	87
25	27	3	3	1	9	78
26	15	3	3	2	12	85
27	17	3	3	3	14	93

The SD was used to indicate the variation among the repeated measurements whereas the mean value of the three repeated observations shown as TABLE 4.4 was considered as the

representative experimental response for DOE and subsequent statistical analysis. This approach minimizes the influence of random experimental variations and improves the accuracy of the developed statistical models.

These results were analysed using MINITAB 19 statistical software to determine the significance of the factors and their interactions on the selected response parameters. The corresponding main effect plots for FT and BT are shown in FIGURE 4.4 and FIGURE 4.5, respectively.

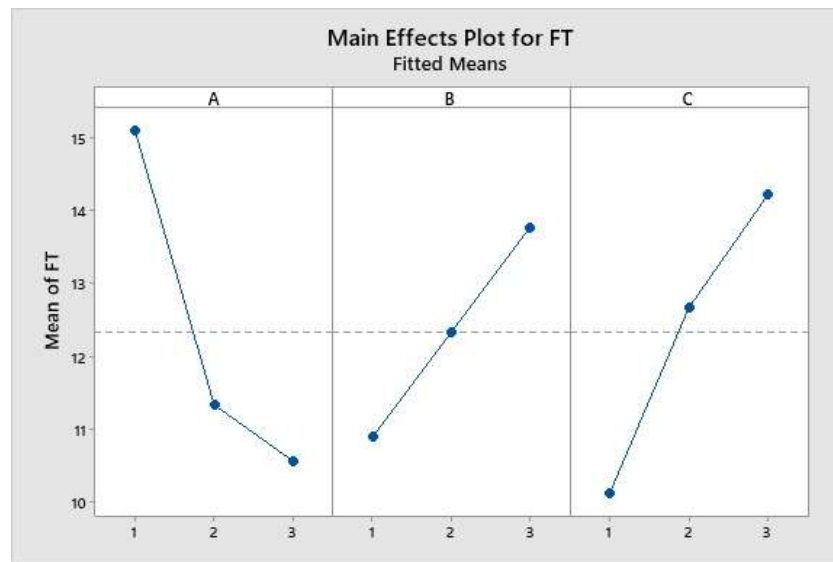


FIGURE 4.4 Main Effective Plots for FT

FIGURE 4.4 illustrates the influence of rib convexity, axial load, and operating speed on FT. It can be observed that FT increases with an increase in axial load and operating speed, whereas it decreases with an increase in rib convexity. The increase in FT with axial load can be attributed to the increase in contact force between the roller ends and the inner-ring rib, which increases the sliding resistance and frictional losses at the rib–roller-end interface. As the axial load increases from 2000 to 6000 kgf, the contact conditions become more severe, resulting in a progressive increase in FT. Similarly, an increase in operating speed from 400 to 600 rpm increases the relative sliding velocity and the frequency of roller–rib interactions. Consequently, greater frictional energy is dissipated within the bearing, leading to an increase in FT.

Conversely, FT decreases with increasing rib convexity. A higher rib convexity provides a more favourable roller end/rib contact geometry, which can reduce the severity of localized

contact and sliding resistance. At low rib convexity, the contact becomes more concentrated, increasing the frictional resistance and consequently the friction torque. This effect is particularly evident at higher axial loads and speeds, where the influence of the rib–roller-end contact becomes more pronounced. The minimum FT was obtained at a rib convexity of 6 μm , axial load of 2000 kgf, and operating speed of 400 rpm.

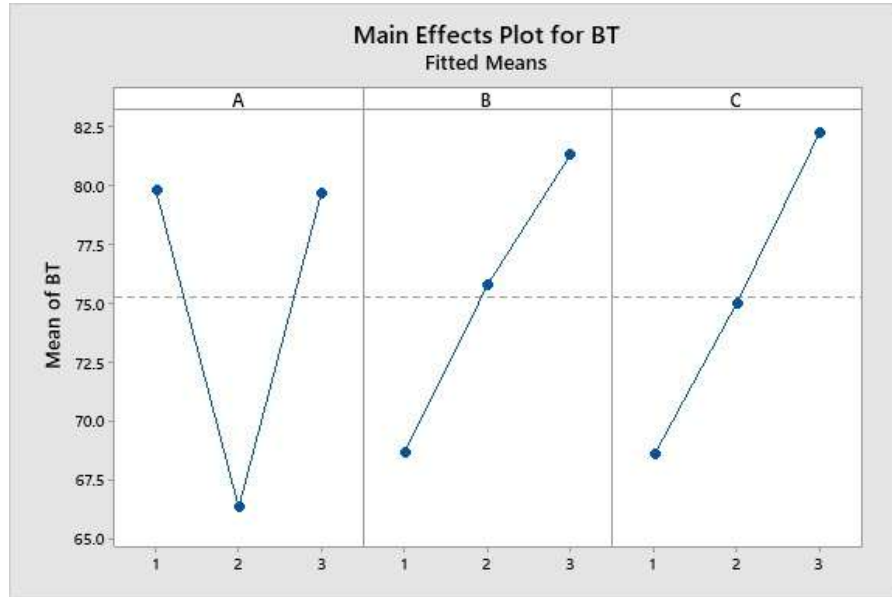


FIGURE 4.5 Main Effective Plots for BT

The main effect plot for BT as shown in FIGURE 4.5 illustrates the influence of rib convexity, axial load, and operating speed on BT. It is observed that BT increases with increasing axial load and operating speed, due to the higher frictional heat generated under increased load and rotational speed. In contrast, the effect of rib convexity is non-linear, with the minimum BT observed at 3 μm rib convexity, while both 0 μm and 6 μm result in comparatively higher temperatures. This indicates that an appropriate rib convexity provides more favourable contact conditions and helps reduce heat generation. The maximum BT was observed at 0 μm rib convexity, 6000 kgf axial load, and 600 rpm, whereas the minimum BT was obtained near 3 μm rib convexity, 2000 kgf axial load, and 400 rpm. Overall, the results indicate that axial load and speed are the dominant factors increasing BT, while rib convexity exhibits an optimum value for minimizing bearing temperature.

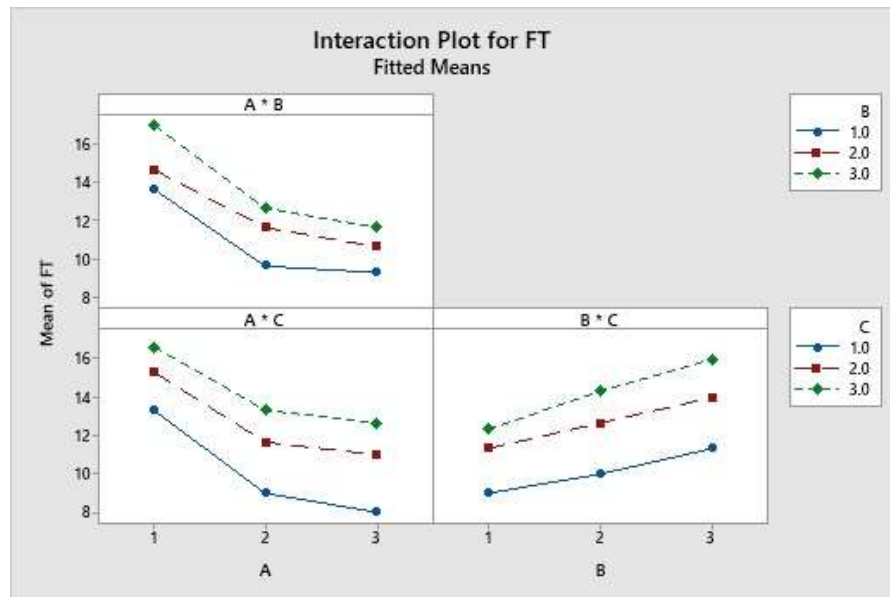


FIGURE 4.6 Interaction Plots for FT

The interaction plots in FIGURE 4.6 presents the interaction effects of rib convexity (A), axial load (B), and operating speed (C) on friction torque (FT). The interaction plots show that the response is influenced by the combined levels of the factors rather than by their individual effects alone.

Interaction between rib convexity and axial load ($A \times B$): The $A \times B$ plot shows that FT decreases as rib convexity increases for all three levels of axial load. However, the decrease is more pronounced at higher axial loads. The highest FT is observed at low rib convexity and high axial load, whereas increasing rib convexity considerably reduces FT. This indicates that the effect of rib convexity becomes more significant under higher axial loading conditions.

Interaction between rib convexity and operating speed ($A \times C$): The $A \times C$ plot also shows a decreasing trend in FT with increasing rib convexity at all speed levels. The reduction in FT is particularly evident when the bearing operates at higher speeds. Thus, low rib convexity combined with high operating speed results in higher friction torque, while higher rib convexity helps reduce FT.

Interaction between axial load and operating speed ($B \times C$): The $B \times C$ plot shows that FT increases with both axial load and operating speed. At each speed level, increasing axial load increases FT, and at each load level, increasing speed also increases FT. The higher FT

values at simultaneous high load and high speed indicate that these operating parameters together produce more severe frictional conditions.

The non-parallel nature of the interaction lines indicates that the effect of one factor changes with the level of another factor, demonstrating the presence of interaction effects. Among the combinations shown, the highest FT occurs under low rib convexity, high axial load, and high operating speed, while increasing rib convexity and reducing the load and speed results in lower FT.

The interaction plots shown in FIGURE 4.7 provide further insight into this phenomenon. The $A \times B$ and $A \times C$ interaction plots demonstrate that the influence of rib convexity on BT remains significant at all load and speed levels. At the intermediate rib convexity of $3 \mu\text{m}$, the contact conditions between the roller end and rib appear to be more favourable, resulting in lower heat generation. However, both lower and higher convexity values produce comparatively higher temperatures due to less favourable contact characteristics and localized stress effects at the roller-rib interface.

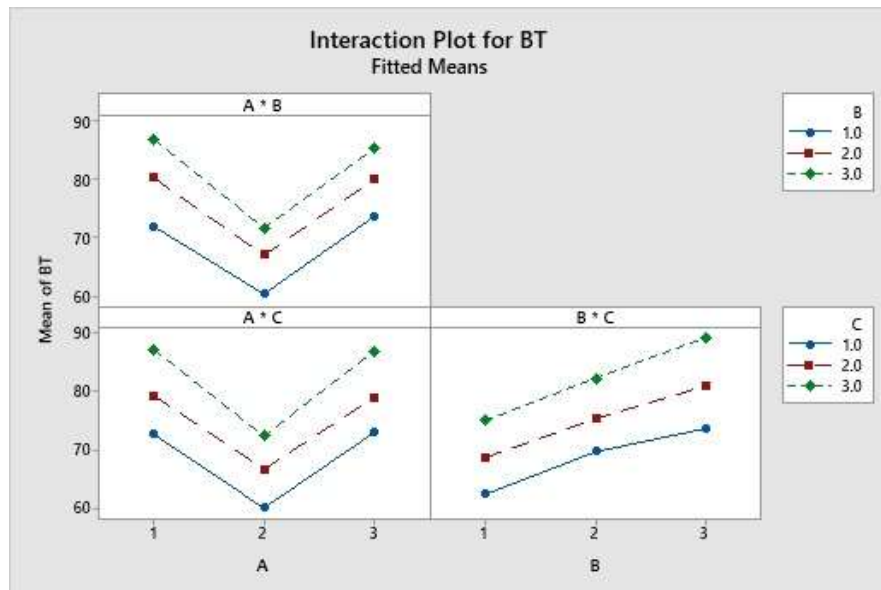


FIGURE 4.7 Interaction Plots for BT

The $B \times C$ interaction plot indicates that BT increases consistently with increasing axial load and operating speed. The nearly parallel but rising trend of the interaction lines suggests that both factors contribute simultaneously to frictional heat generation. The highest

temperatures are observed under conditions of high load and high speed, where the combined effects of increased contact pressure and frictional energy dissipation become significant.

Overall, the results indicate that rib convexity, axial load, and operating speed have significant effects on the thermal and tribological performance of tapered roller bearings. While higher rib convexity reduces FT, an intermediate rib convexity of approximately 3 μm provides the lowest BT. Furthermore, increasing axial load and operating speed consistently increase both FT and BT. These observations highlight the importance of optimizing rib geometry together with operating conditions to achieve improved bearing performance.

4.5 Development of FFD Regression Model for FT and BT

TRBs operate under complex loading and kinematic conditions where multiple parameters simultaneously influence their thermal and tribological performance. Therefore, establishing a mathematical relationship between the process parameters and the response variables is essential for performance prediction, optimization, and process control.

Various predictive modelling techniques such as statistical models, machine learning models, artificial neural networks and soft computing approaches are available for analysing engineering systems. Among these techniques, regression analysis is one of the most widely used statistical methods for developing mathematical relationships between independent variables and response parameters. A regression model enables the prediction of system responses within the selected experimental domain and provides insight into the significance of individual factors and their interactions (McCarthy et al., 2022; Montgomery, 2010).

4.5.1 FFD Regression Models

Since the present study involves three independent variables and potential interaction effects among them, a second-order multiple regression model (quadratic model) was adopted. Compared to first-order models, quadratic regression models provide better prediction capability and can effectively represent curvature and interaction effects within the experimental region. The generalized second-order regression equation is expressed as Equation 4.1.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \dots + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \dots \quad 4.1$$

where y represents the response variable, x_i denotes the independent variables, and β_0 , β_i , β_{ij} , and β_{ii} are the regression coefficients corresponding to the intercept, linear, interaction, and quadratic terms, respectively (Montgomery, 2013).

Based on the regression analysis, the following second-order regression models were developed for predicting FT as Equation 4.2 and BT as Equation 4.3 as functions of rib convexity (A), axial load (B), and operating speed (C).

$$\begin{aligned} \text{FT} = & 14.89 - 8.444 A + 1.278 B + 2.722 C + 1.500 A * A \\ & - 0.500 C * C - 0.250 A * B + 0.333 A * C \\ & + 0.333 B * C \end{aligned} \quad 4.2$$

$$R^2 = 98.87\%, \quad R^2(\text{adj}) = 98.27\%$$

$$\begin{aligned} \text{BT} = & 91.37 - 51.61 A + 9.61 B + 4.11 C + 13.389 A * A \\ & - 0.778 B * B + 0.389 C * C - 0.833 A * B \\ & - 0.167 A * C + 0.750 B * C \end{aligned} \quad 4.3$$

$$R^2 = 99.36\%, \quad R^2(\text{adj}) = 95.15\%$$

The coefficient of determination (R^2) indicates the proportion of variation in the response explained by the regression model, whereas the adjusted coefficient of determination $R^2(\text{adj})$ accounts for the number of predictors included in the model and therefore provides a more reliable measure of model adequacy in multiple regression analysis. Unlike R^2 , which may increase with the addition of new predictors irrespective of their significance, $R^2(\text{adj})$ increases only when the additional terms improve the predictive capability of the model.

The developed FT model yielded R^2 and $R^2(\text{adj})$ values of 98.87% and 98.27%, respectively, while the BT model produced R^2 and $R^2(\text{adj})$ values of 99.36% and 95.15%, respectively. The high values of both statistical indicators demonstrate excellent agreement between the predicted and experimental results. Therefore, the developed regression models can be considered statistically significant and sufficiently accurate for predicting FT and BT within the selected experimental domain.

4.5.2 Normal Probability Plots and Anderson–Darling (AD) Test

The experimental data obtained from the FFD were analysed using Minitab19 statistical software. After developing the regression models, residual analysis was performed to verify the adequacy of the fitted models. The normal probability plots of residuals for FT and BT are shown in FIGURE 4.8 and FIGURE 4.9, respectively.

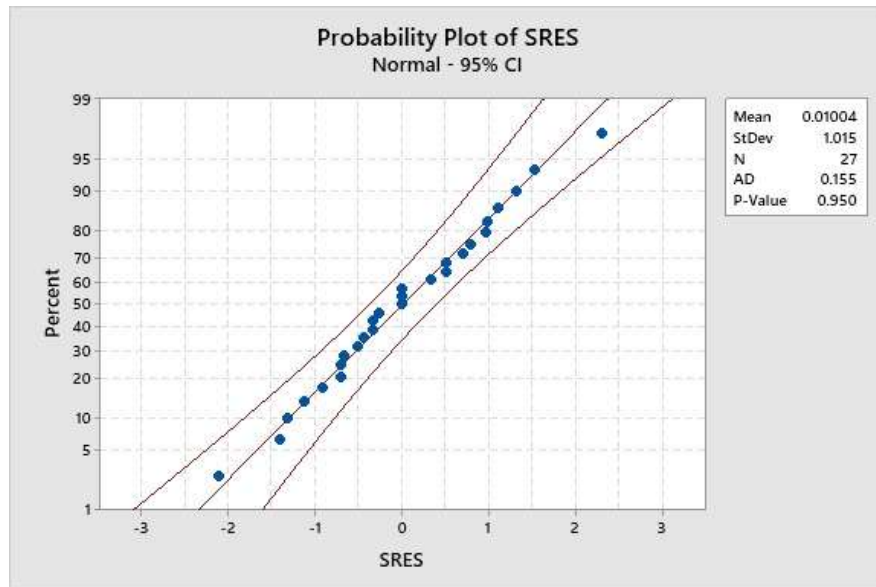


FIGURE 4.8 Normal Probability Plot for Residuals for FT

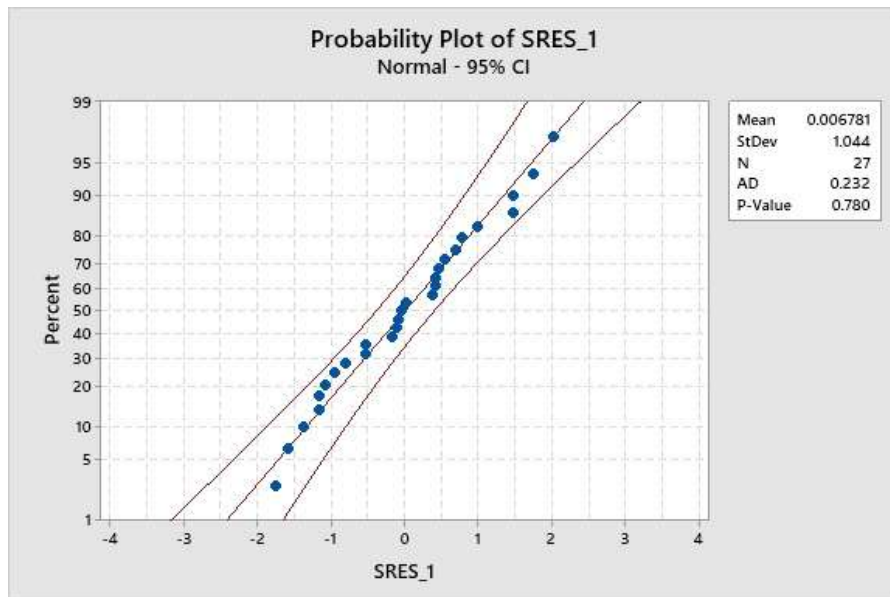


FIGURE 4.9 Normal Probability Plot for Residuals for BT

The residuals exhibited an approximately linear distribution, indicating that the assumptions of normality were satisfied. Furthermore, the Anderson–Darling (AD) test statistics yielded p-values greater than 0.05 for both responses, confirming the validity of the normality assumption and supporting the adequacy of the developed models.

4.5.3 Residuals Versus Fits Plot

The residuals versus fitted values for FT show that the residual points are distributed on both sides of the zero reference line without any clear systematic pattern. The standardized residuals are approximately within the range of -2.1 to $+2.3$, indicating that the deviations between the experimental and fitted values are relatively small. The points appear to be randomly scattered across the fitted-value range, with no distinct curvature or funnel-shaped pattern. This indicates that there is no strong evidence of non-linearity or non-constant variance in the FT model. Therefore, the residual plot supports the adequacy of the developed regression model for predicting friction torque.

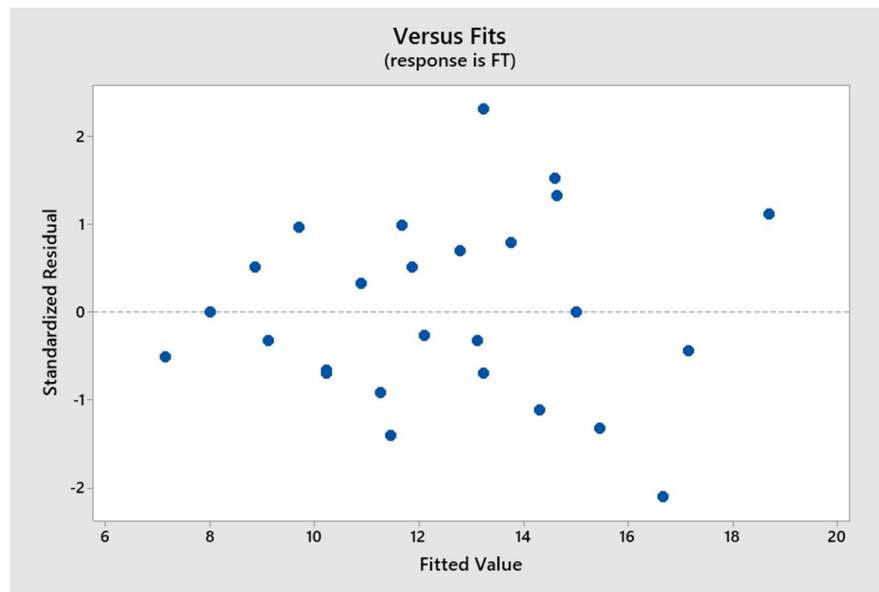


FIGURE 4.10 Residual Vs Fits for FT

The residuals versus fitted values for BT also show a generally random distribution around the zero line. The standardized residuals are approximately within -1.8 to $+2.1$, and no individual point shows an unusually large residual.

There is no clear increasing or decreasing trend, curvature, or funnel-shaped distribution of the residuals over the fitted-value range. This indicates that the model does not exhibit a significant systematic error or variance-related problem. Hence, the residual plot indicates that the BT regression model provides an adequate representation of the experimental data.

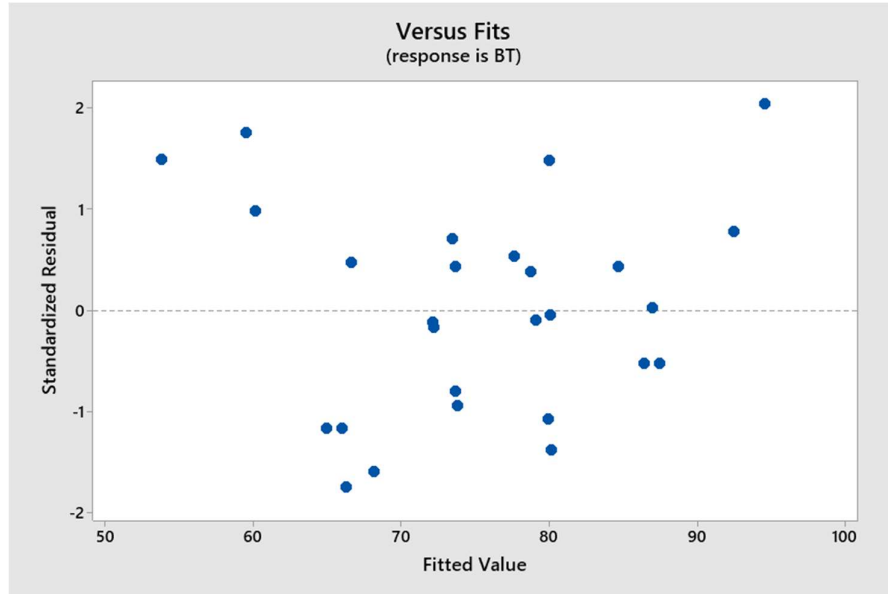


FIGURE 4.11 Residual Vs Fits for BT

Overall, both plots show randomly distributed residuals around zero with no prominent systematic pattern. This indicates that the regression models for FT and BT are reasonably appropriate and that the assumptions of independence, linearity, and approximately constant variance of residuals are reasonably satisfied.

4.5.4 Analysis of Variance (ANOVA)

Analysis of Variance (ANOVA) is a powerful statistical technique widely used in engineering and mechanical experimentation to evaluate the significance of process parameters and their interactions on selected response variables. ANOVA helps in identifying the factors that significantly influence the responses and assessing the adequacy of the developed regression models (Montgomery, 2010, 2013). In the present study, ANOVA was employed to investigate the effects of rib convexity (A), axial load (B), and operating speed (C) on FT and BT. To improve the predictive capability of the models, linear, quadratic, and two-factor interaction terms were included in the analysis.

The ANOVA results for FT and BT are presented in TABLE 4.5 and TABLE 4.6 respectively. In these tables, DF represents the Degree of Freedom, Adj SS denotes the Adjusted Sum of Squares, and Adj MS represents the Adjusted Mean Square.

TABLE 4.5 ANOVA Table for FT

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	9	225.417	25.0463	164.82	0.000
A	1	21.042	21.0419	138.47	0.000
B	1	0.482	0.4818	3.17	0.093
C	1	2.187	2.1867	14.39	0.001
A x A	1	13.500	13.5000	88.84	0.000
B x B	1	0.000	0.0000	0.00	1.000
C x C	1	1.500	1.5000	9.87	0.006
A x B	1	0.750	0.7500	4.94	0.040
A x C	1	1.333	1.3333	8.77	0.009
B x C	1	1.333	1.3333	8.77	0.009
Error	17	2.583	0.1520		
Total	26	228.000			

The F-value indicates the ratio of model variance to error variance and is used to evaluate the significance of the model terms. A higher F-value generally indicates a more significant effect of the corresponding factor on the response. The significance of each term is determined using the associated p-value, with a significance level of $\alpha = 0.05$.

For both FT and BT, the overall regression models exhibit p-values less than 0.05, indicating that the developed models are statistically significant and adequately explain the variability present in the experimental data. Terms having p-values less than 0.05 are considered significant contributors to the model, whereas terms with p-values greater than 0.05 are regarded as statistically insignificant.

TABLE 4.6 ANOVA Table for BT

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	9	2658.08	295.34	293.58	0.000
A	1	786.01	786.01	781.33	0.000
B	1	27.26	27.26	27.10	0.000
C	1	4.99	4.99	4.96	0.040
A x A	1	1075.57	1075.57	1069.17	0.000
B x B	1	3.63	3.63	3.61	0.075
C x C	1	0.91	0.91	0.90	0.356
A x B	1	8.33	8.33	8.28	0.010
A x C	1	0.33	0.33	0.33	0.572
B x C	1	6.75	6.75	6.71	0.019
Error	17	17.10	1.01		
Total	26	2675.19			

For the FT model as shown in TABLE 4.5, the axial load term B and the quadratic term B² exhibit p-values greater than 0.05 and are therefore considered statistically insignificant. Similarly, for the BT model as shown in TABLE 4.6, the quadratic terms B² and C², along with the interaction term A×C, are found to be insignificant. These insignificant terms may be omitted during model refinement without substantially affecting the predictive capability of the regression models.

4.5.5 Pareto Charts for Standardized Effects

The significance of the individual model terms is further illustrated through the Pareto charts of standardized effects shown in FIGURE 4.12 and FIGURE 4.13 for FT and BT, respectively. In the Pareto charts, the vertical reference line represents the critical significance threshold. The factors whose standardized effects lie beyond this reference line are considered statistically significant, whereas those falling below the threshold are insignificant. The observations obtained from the Pareto charts are consistent with the ANOVA results.

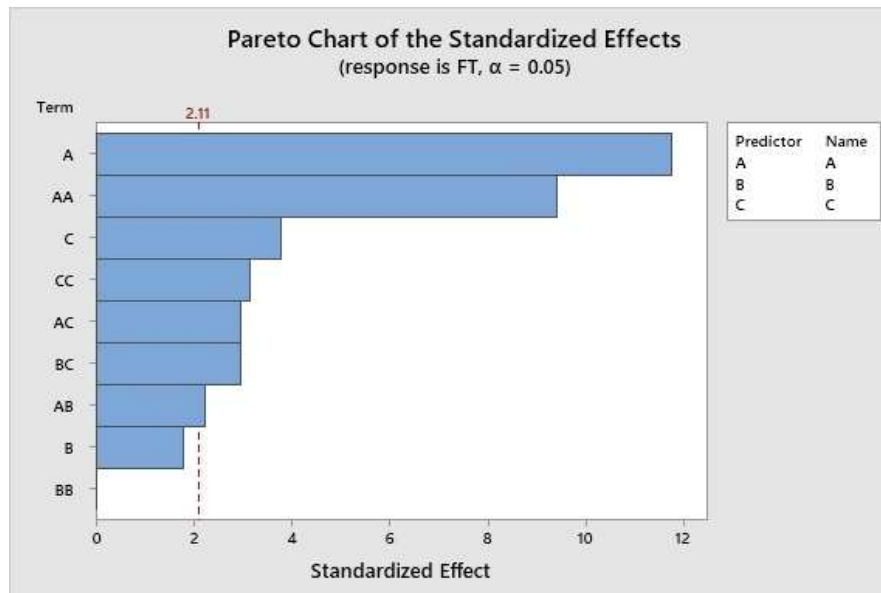


FIGURE 4.12 Pareto Chart of the Standardized Effect for FT

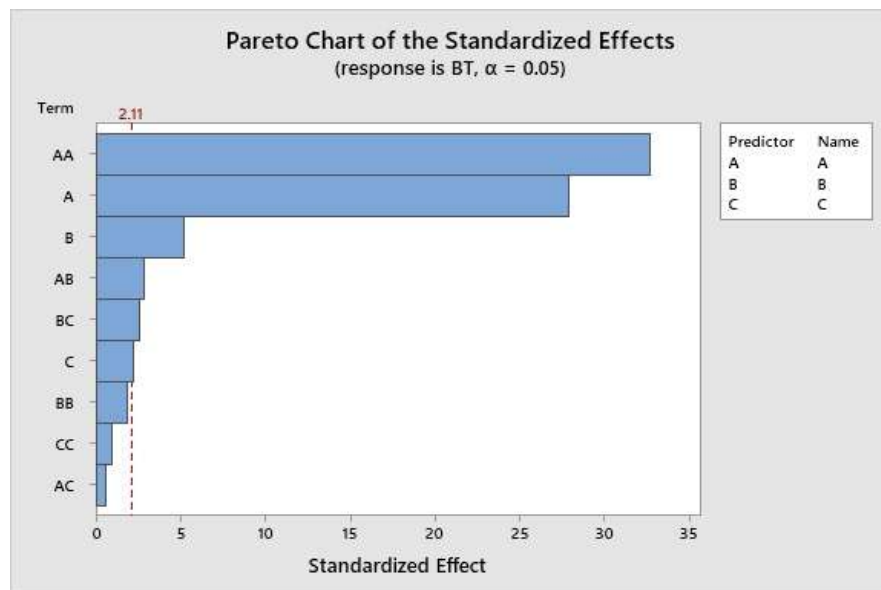


FIGURE 4.13 Pareto Chart of the Standardized Effect for BT

4.5.6 Comparison of Experimental Results with Regression Model Fits

Further validation of the regression models was carried out by comparing the experimental observations with the predicted values generated by the regression equations. The comparison is presented in TABLE 4.7.

TABLE 4.7 Comparison Of Experimental Results With Regression Model Fits Along Percentage Errors

Sr. No	Factors			Experiment		Regression		Percentage Error	
	A	B	C	FT	BT	FT_FITS	BT_FITS	FT	BT
1.	1	1	1	12	65	11.8611	66.2315	1.16	1.89
2.	1	1	2	14	72	13.7500	72.0926	1.79	0.13
3.	1	1	3	15	79	14.6389	78.7315	2.41	0.34
4.	1	2	1	13	74	13.2222	73.4259	1.71	0.78
5.	1	2	2	15	80	15.4444	80.0370	2.96	0.05
6.	1	2	3	16	87	16.6667	87.4259	4.17	0.49
7.	1	3	1	15	79	14.5833	79.0648	2.78	0.08
8.	1	3	2	17	86	17.1389	86.4259	0.82	0.50
9.	1	3	3	19	96	18.6944	94.5648	1.61	1.49
10.	2	1	1	8	55	8.0000	53.7870	0.00	2.21
11.	2	1	2	10	61	10.2222	59.4815	2.22	2.49
12.	2	1	3	11	65	11.4444	65.9537	4.04	1.47
13.	2	2	1	9	61	9.1111	60.1481	1.23	1.40
14.	2	2	2	12	67	11.6667	66.5926	2.78	0.61
15.	2	2	3	14	73	13.2222	73.8148	5.56	1.12
16.	2	3	1	10	64	10.2222	64.9537	2.22	1.49
17.	2	3	2	13	72	13.1111	72.1481	0.85	0.21
18.	2	3	3	15	79	15.0000	80.1204	0.00	1.42
19.	3	1	1	7	67	7.1389	68.1204	1.98	1.67
20.	3	1	2	10	73	9.6944	73.6481	3.06	0.89
21.	3	1	3	11	81	11.2500	79.9537	2.27	1.29
22.	3	2	1	8	74	8.0000	73.6481	0.00	0.48
23.	3	2	2	11	79	10.8889	79.9259	1.01	1.17
24.	3	2	3	13	87	12.7778	86.9815	1.71	0.02
25.	3	3	1	9	78	8.8611	77.6204	1.54	0.49
26.	3	3	2	12	85	12.0833	84.6481	0.69	0.41
27.	3	3	3	14	93	14.3056	92.4537	2.18	0.59

The maximum percentage prediction errors for FT and BT were found to be 5.56% and 2.49%, respectively, while the minimum errors were 0% and 0.02%, respectively. These relatively low prediction errors indicate good agreement between the experimental and predicted values.

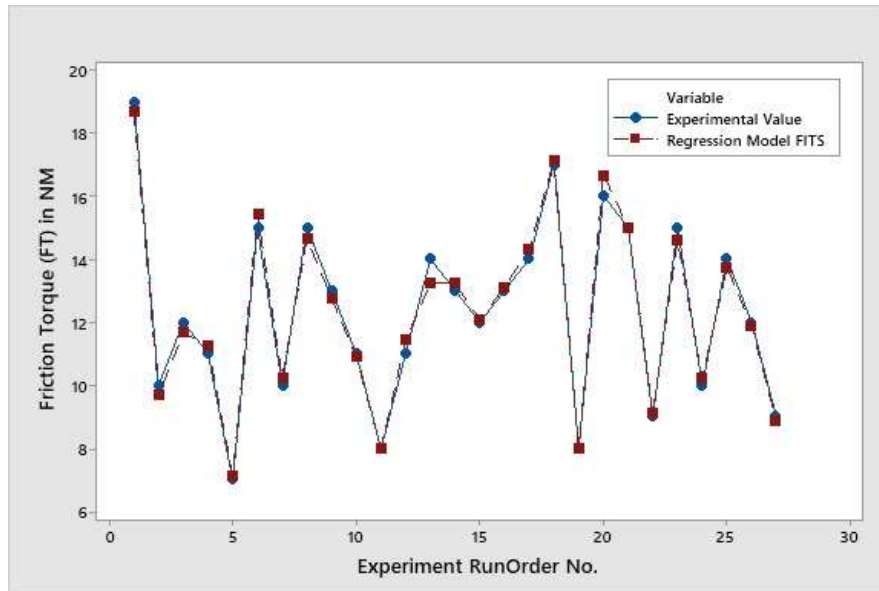


FIGURE 4.14 FT Experiment Results Vs. Regression Model Fits

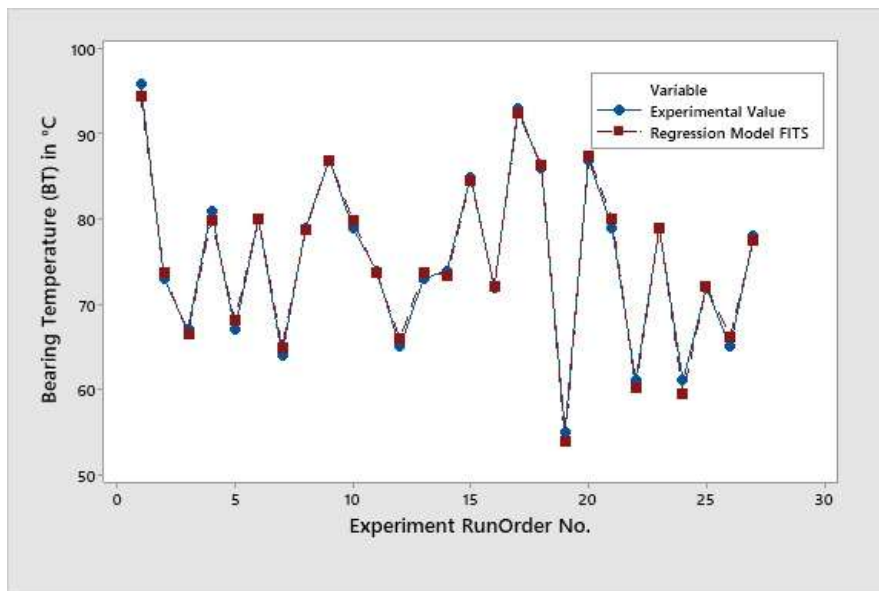


FIGURE 4.15 BT Experiment Results Vs. Regression Model Fits

The graphical comparison between experimental and predicted responses is shown in FIGURE 4.14 and FIGURE 4.15 for FT and BT, respectively. In these figures, the experimental values and regression model predictions exhibit a close correlation throughout the experimental domain. The high coefficient of determination (R^2), satisfactory adjusted coefficient of determination (R^2_{adj}), normally distributed residuals, and low prediction errors collectively confirm that the developed regression models are statistically adequate and can reliably predict the FT and BT within the selected range of operating conditions.

4.6 Response Surface Methodology (RSM)

Although the FFD provides comprehensive information regarding the main and interaction effects of process parameters, a more detailed understanding of the response behaviour within the experimental region is often required. For this purpose, RSM was employed in the present study to establish mathematical relationships between the process parameters and response variables and to investigate their combined effects through response surface analysis. RSM is one of the most widely used Design of Experiments (DOE) techniques for process development, product design, modeling, and multi response optimization (MRO) studies (Khuri & Cornell, 1996; Passari et al., 2022). It enables researchers to systematically plan experiments, evaluate the effects of multiple variables simultaneously, reduce the number of experimental trials, and develop predictive mathematical models for analysing complex engineering systems (Anderson-Cook et al., 2009; Dean et al., 2017; Khuri & Cornell, 1996; Khuri & Siuli Mukhopadhyay, 2010; Ziegel, 1997).

The primary objective of RSM is to determine the functional relationship between the response variable and the independent process parameters. This relationship is generally represented using a second-order polynomial model, which is capable of describing both linear and nonlinear behaviour of the system. The inclusion of interaction and quadratic terms allows the model to accurately predict the response and identify optimum operating conditions within the selected experimental domain.

The generalized second-order response surface model can be expressed as Equation 4.4:

$$y = \beta_0 + \sum \beta_i x_i + \sum \beta_{ij} x_i x_j + \sum \beta_{ii} x_i^2 \quad 4.4$$

where y represents the response variable, x_i denotes the independent variables, and β_0 , β_i , β_{ij} , and β_{ii} are the regression coefficients corresponding to the intercept, linear, interaction, and quadratic terms, respectively (Montgomery, 2013).

RSM designs are commonly classified into two major categories: Central Composite Design CCD and BBD. Both designs are capable of developing second-order predictive models; however, the BBD requires fewer experimental runs than CCD for the same number of factors and avoids experimental combinations at extreme corner points of the design space. Consequently, BBD is considered efficient, economical, and suitable for engineering investigations involving multiple process parameters (Fisher, 1974), (Bhivane et al., 2024;

Dean et al., 2017; Yıldız et al., 2023). In the present study, the BBD was selected to develop response surface models for FT and BT. The design facilitates the evaluation of linear, quadratic, and interaction effects of rib convexity, axial load, and operating speed while minimizing the number of experimental trials. Furthermore, the developed response surfaces provide a graphical representation of the influence of process parameters on the responses and serve as the basis for the optimization studies presented in the subsequent chapter.

4.6.1 Response Surface Methodology based Box–Behnken Design (RSM BBD)

In the present study, Box–Behnken Design (BBD) was employed as the experimental design under the Response Surface Methodology (RSM) framework. RSM was used for developing quadratic models, analysing factor effects, and optimization, whereas BBD provided the experimental combinations required for RSM analysis. Therefore the RSM BBD was employed to develop predictive models and investigate the combined effects of the selected factors on bearing performance. The three input parameters considered in the present study were rib convexity (A), axial load (B), and rotational speed (C). The coded and actual levels of these factors are presented in TABLE 4.8. For three factors at three levels, the design generated a set of experimental combinations as shown in TABLE 4.9. The experiments were conducted according to the design matrix, and the corresponding responses, namely FT and BT, were recorded.

TABLE 4.8 Factors and Their Selected Levels

Factors	Code	Level			Unit
		-1	0	1	
Rib Convexity	A	0	3	6	μ
Axial Load	B	2000	4000	6000	kgf
Bearing Speed	C	400	500	600	rpm

TABLE 4.9 Box Behnken Design With Coded Factors With Responses

Run Order	A	B	C	Friction Torque (Nm)	Bearing Temperature (°C)
1	-1	-1	0	14	72
2	1	-1	0	10	73
3	-1	1	0	17	86
4	1	1	0	12	85

5	-1	0	-1	13	74
6	1	0	-1	8	74
7	-1	0	1	16	87
8	1	0	1	13	87
9	0	-1	-1	8	55
10	0	1	-1	10	64
11	0	-1	1	11	65
12	0	1	1	15	79
13	0	0	0	12	78
14	0	0	0	12	74
15	0	0	0	12	76

4.6.2 RSM BBD Regression Model

The experimental data obtained from the BBD were analysed further in MINITAB 19 to develop mathematical models for the selected responses, namely FT and BT.

TABLE 4.10 RSM BBD Regression Model

Responses	Correlation	Equation No
Friction Torque (FT)	$= -12.63 - 2.292A - 0.000187B + 0.0925C + 0.1528A^2 - 0.000000B^2 - 0.000088C^2 - 0.000042AB + 0.001667AC + 0.000002BC$	4.5
Bearing Temperature (BT)	$= -80.6 - 5.58A + 0.01194B + 0.4763C + 0.9861A^2 - 0.000001B^2 - 0.000437C^2 - 0.000083AB - 0.000000AC + 0.000006BC$	4.6

The Response Surface Regression (RSR) models establish a quantitative relationship between the process parameters and the corresponding responses. These models represent the influence of rib convexity (A), axial load (B), and rotational speed (C), including their linear, quadratic, and interaction effects. The developed regression equations for FT as shown in Equation 4.5 and BT as shown in Equation 4.6 are presented as TABLE 4.10. The coefficients of the regression equations indicate the magnitude and direction of the influence of each factor on the responses. Positive coefficients indicate that the corresponding

parameter increases the response, whereas negative coefficients indicate a decreasing effect. The quadratic and interaction terms reveal the nonlinear behaviour and combined influence of the process parameters on FT and BT.

These regression models provide an effective means of predicting response values within the experimental domain and serve as the basis for generating response surface plots, contour plots, and optimization studies. The developed models also facilitate a better understanding of the relationship between the operating parameters and the performance characteristics of TRBs under various operating conditions.

4.6.3 RSM BBD Based Analysis of variance (ANOVA)

TABLE 4.11 for FT, the developed model exhibits an R^2 value of 99.74%, indicating that the model accounts for nearly all the variability present in the experimental response. The adjusted R^2 and predicted R^2 values of 99.27% and 95.85%, respectively, further demonstrate the robustness and predictive capability of the model. The ANOVA results reveal that rib convexity (A), axial load (B), and rotational speed (C) significantly influence FT, as evidenced by their p-values being less than 0.05. In contrast, the interaction effects (AB, AC, and BC) and the quadratic terms associated with axial load (B^2) and speed (C^2) are statistically insignificant, indicating that their contribution to FT variation within the investigated range is minimal.

TABLE 4.11 for BT indicate excellent model adequacy, with an R^2 value of 99.21%, an adjusted R^2 value of 97.79%, and a predicted R^2 value of 96.75%. These statistical indicators confirm the strong agreement between the experimental and predicted results. Among the investigated parameters, axial load (B) and rotational speed (C) have a significant effect on BT, as their p-values are below the significance level of 0.05. The remaining terms, having p-values greater than 0.05, are considered statistically insignificant and therefore have a negligible influence on the response.

TABLE 4.11 RSM Analysis of Variance

Source	Friction Torque			Bearing Temperature	
	DF	SS	P-Value	SS	P-Value
Model	9	96.1500	0.000	1161.68	0.000
Linear	3	83.2500	0.000	625.25	0.000

A	1	36.1250	0.000	0.00	1.000
B	1	15.1250	0.000	300.13	0.000
C	1	32.0000	0.000	325.13	0.000
Square	3	10.6500	0.000	529.18	0.000
A x A	1	6.9808	0.000	290.83	0.000
B x B	1	0.0577	0.332	127.44	0.000
C x C	1	2.8269	0.001	70.67	0.002
2-Way Interaction	3	2.2500	0.006	7.25	0.369
A x B	1	0.2500	0.076	1.00	0.495
A x C	1	1.0000	0.007	0.00	1.000
B x C	1	1.0000	0.007	6.25	0.125
Error	5	0.2500		9.25	
Total	14	96.4000		1170.93	
R-square		99.74%		99.21%	
R-square (adj)		99.27%		97.79%	
R-square (pred)		95.85%		96.75%	

Overall, the ANOVA results validate the adequacy of the developed response surface models and demonstrate their effectiveness in describing the relationship between the operating parameters and the performance responses. The high coefficient of determination values and strong predictive capability confirm the suitability of the models for further response analysis and optimization studies.

4.6.4 Comparison of Experimental Results with RSM Model

TABLE 4.12 presents a comparison between the experimental values and the values predicted by the developed RSM regression models for FT and BT, along with the corresponding percentage errors. The comparison demonstrates a close agreement between the experimental and predicted results, indicating the adequacy of the developed models. The maximum prediction error observed for FT and BT is 2.08% and 2.70%, respectively,

which is well within the acceptable limit of 5%. These low error values confirm the accuracy and reliability of the regression models in predicting the performance responses within the investigated operating range.

TABLE 4.12 Comparison of Experimental Results With RSM Regression Model Along Percentage Errors

Run	Experimental Results		RSM Model		Percentage Error	
	FT	BT	FT	BT	FT	BT
1	14	72	13.750	72.375	1.79	0.52
2	10	73	10.000	73.375	0.00	0.51
3	17	86	17.000	85.625	0.00	0.44
4	12	85	12.250	84.625	2.08	0.44
5	13	74	13.125	74.125	0.96	0.17
6	8	74	7.875	74.125	1.56	0.17
7	16	87	16.125	86.875	0.78	0.14
8	13	87	12.875	86.875	0.96	0.14
9	8	55	8.125	54.500	1.56	0.91
10	10	64	9.875	64.250	1.25	0.39
11	11	65	11.125	64.750	1.14	0.38
12	15	79	14.875	79.500	0.83	0.63
13	12	78	12.000	76.000	0.00	2.56
14	12	74	12.000	76.000	0.00	2.70
15	12	76	12.000	76.000	0.00	0.00

FIGURE 4.16 and FIGURE 4.17 illustrates the graphical comparison of the experimental and predicted response values for FT and BT respectively. It can be observed that the predicted values closely follow the experimental data, further validating the suitability of the developed RSM. The strong agreement between the experimental observations and model predictions confirms that the regression equations can be effectively utilized for response prediction and subsequent optimization studies.

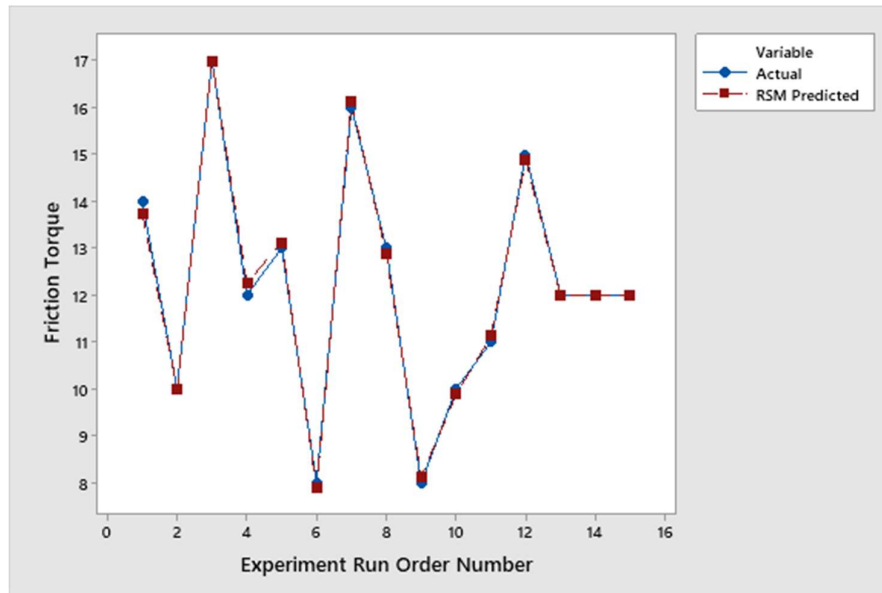


FIGURE 4.16 Graphical Comparison of The Experimental And Predicted Responses for FT

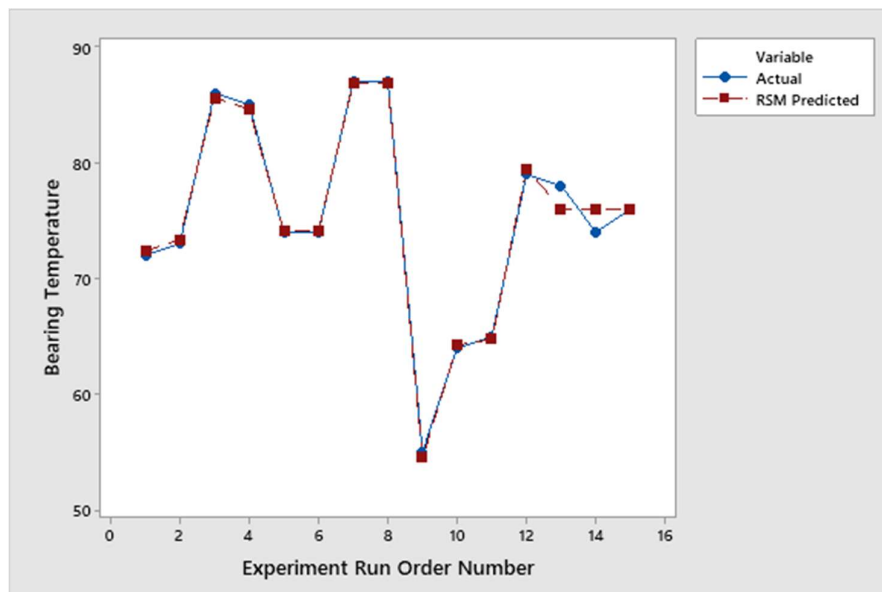


FIGURE 4.17 Graphical Comparison of The Experimental And Predicted Responses for BT

4.6.5 Interaction Effects Between Input and Output Parameters

The RSM was employed to investigate the individual and combined effects of rib convexity (A), axial load (B), and rotational speed (C) on the response variables, namely FT and BT. The main effect plots, contour plots, and response surface plots generated using MINITAB 19 are presented in this section.

FIGURE 4.18 and FIGURE 4.19 show the main effect plots for FT and BT, respectively. These plots illustrate the influence of individual process parameters on the responses. It can be observed that FT increases continuously with increasing axial load and rotational speed. In contrast, FT decreases with increasing rib convexity and attains its minimum value near the highest convexity level of $6\text{ }\mu\text{m}$. This behaviour may be attributed to the reduction in localized contact and sliding resistance at the roller-end and rib interface. For BT, a different trend is observed. The temperature decreases initially with increasing rib convexity and reaches a minimum around the centre level of $3\text{ }\mu\text{m}$, after which it increases again. This indicates the existence of an optimum rib profile that minimizes heat generation at the rib–roller contact region. Furthermore, BT increases significantly with increases in both axial load and rotational speed due to higher frictional energy generation under severe operating conditions.

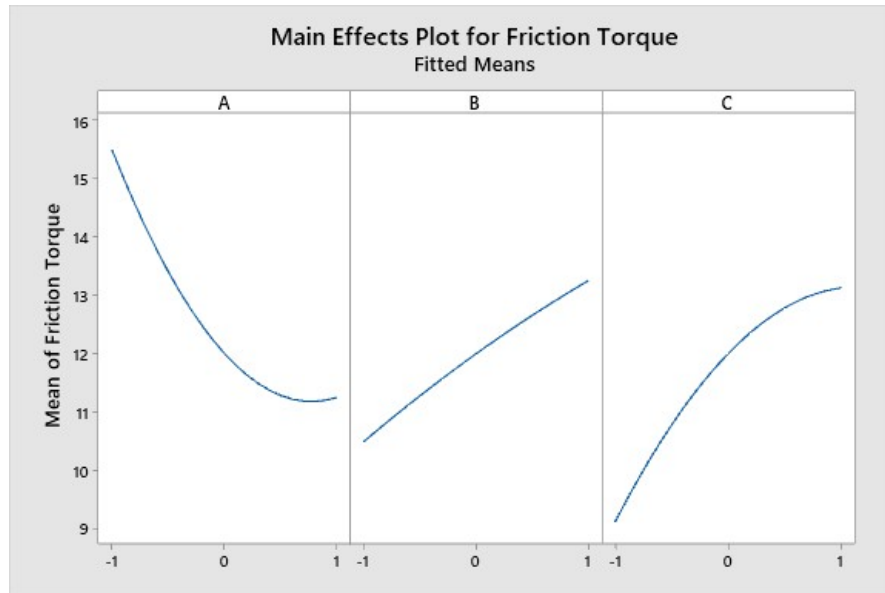


FIGURE 4.18 Main Effects Plot for FT

The interaction effects between the process parameters are illustrated through contour plots shown as FIGURE 4.20 to FIGURE 4.23. FIGURE 4.20 and FIGURE 4.21 represent the contour plots for FT as a function of rib convexity–axial load and rib convexity–speed combinations, respectively. The contour patterns reveal that FT increases with increasing axial load and speed, whereas it decreases with increasing rib convexity. The darker regions correspond to higher FT values, while the lighter regions indicate lower FT. The minimum FT region is observed at higher rib convexity levels combined with lower axial load and speed conditions. FIGURE 4.22 and FIGURE 4.23 show the contour plots for BT with

respect to rib convexity–axial load and rib convexity–speed interactions. The contour distribution indicates that the lowest temperature zone occurs near the rib convexity of 3 μm . As axial load and rotational speed increase, the contour bands shift toward higher temperature regions, demonstrating the strong influence of these parameters on heat generation within the bearing.

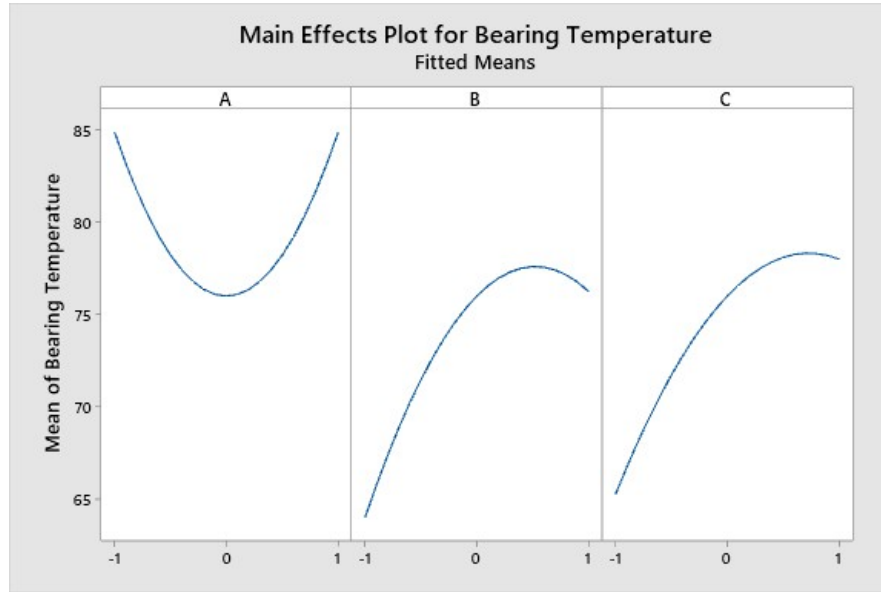


FIGURE 4.19 Main Effect Plot for BT

To further visualize the interaction effects and response behaviour, three-dimensional response surface plots are presented in FIGURE 4.24 to FIGURE 4.27. The surface plots for FT clearly indicate as shown in FIGURE 4.24 and FIGURE 4.25 that the response increases with increasing axial load and rotational speed, particularly at lower rib convexity levels. The highest FT values are observed near 0 μm rib convexity combined with high load and speed conditions. Conversely, the minimum FT region is obtained at higher rib convexity

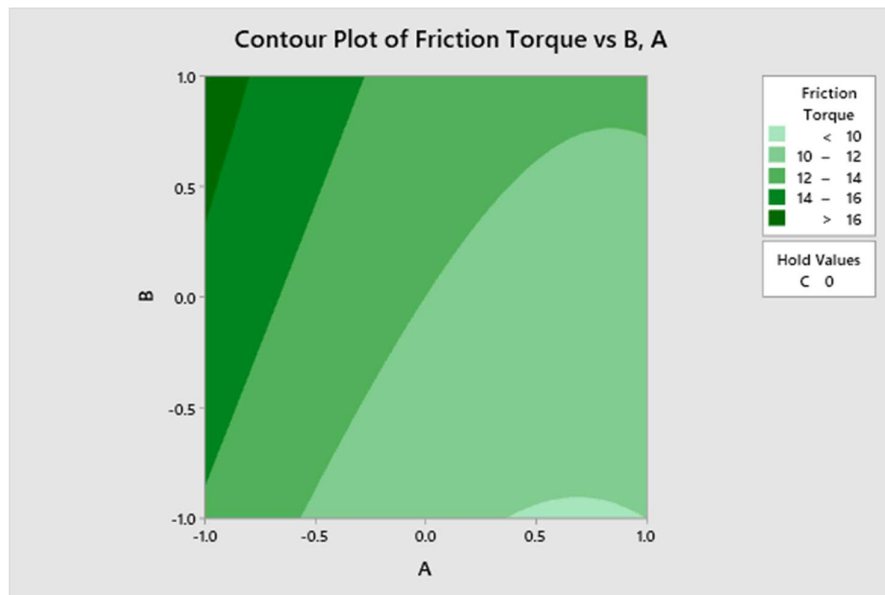


FIGURE 4.20 Contour Plot of FT vs B, A

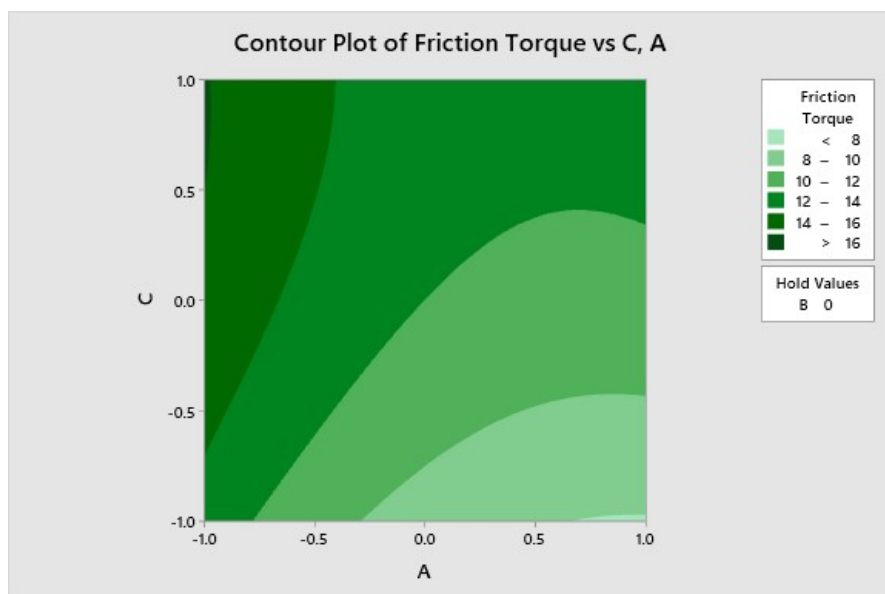


FIGURE 4.21 Contour Plot of FT vs C, A

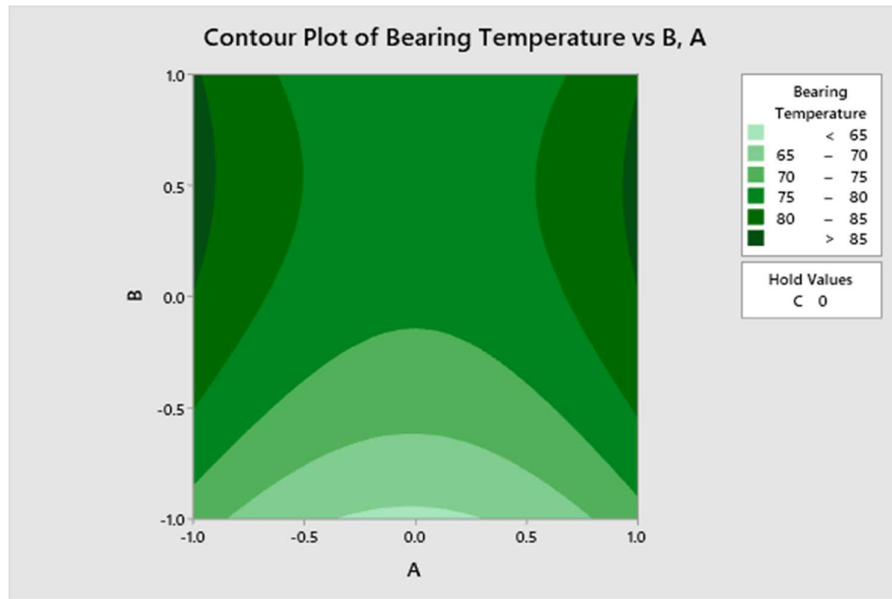


FIGURE 4.22 Contour Plot of BT vs B, A

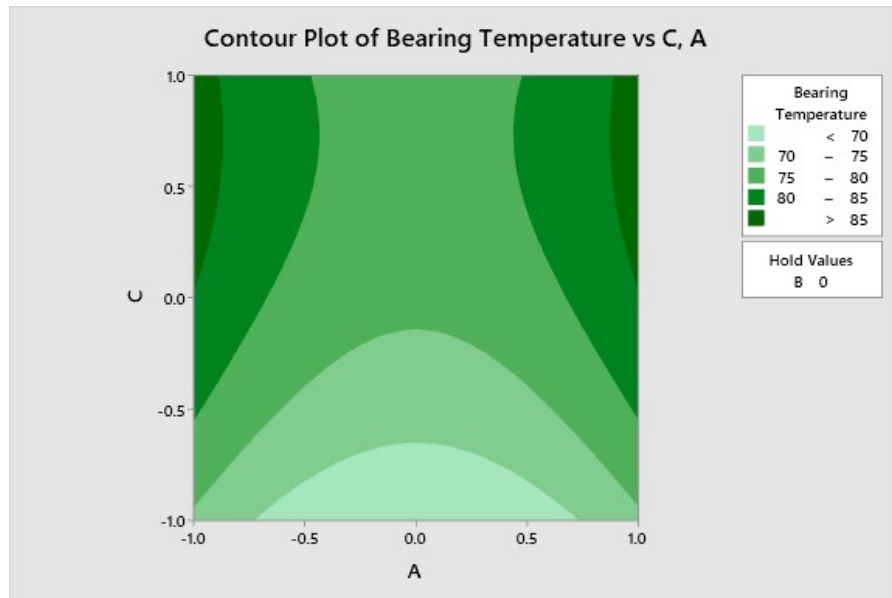


FIGURE 4.23 Contour Plot of BT vs C, A

levels with lower operating loads and speeds. Similarly, the response surface plots for BT demonstrate as shown in FIGURE 4.26 and FIGURE 4.27 that the minimum temperature occurs near the intermediate rib convexity level of 3 μm . The temperature rises considerably with increasing axial load and rotational speed, resulting in steeper response surfaces at

higher operating conditions. The curved nature of the response surfaces confirms the presence of quadratic effects and interactions among the process variables.

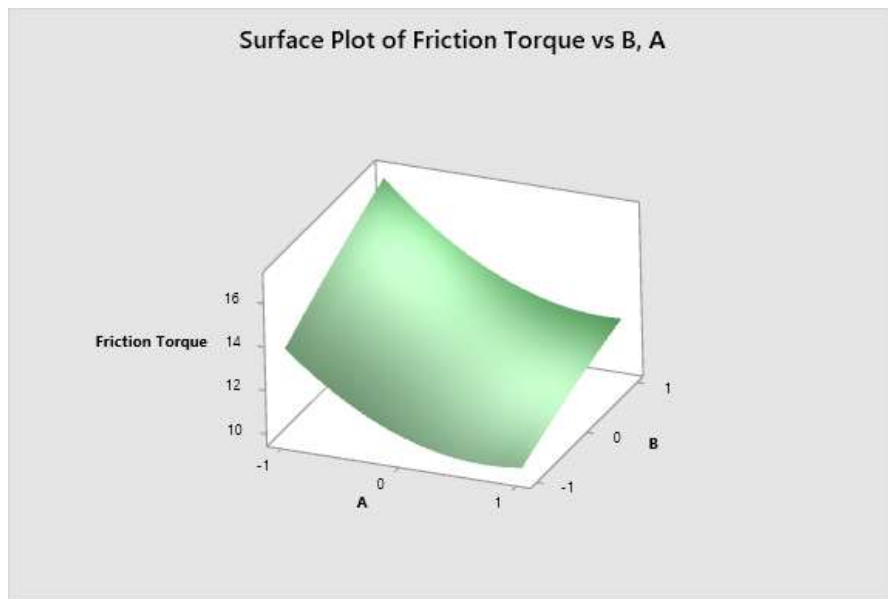


FIGURE 4.24 Surface Plot of FT vs B, A

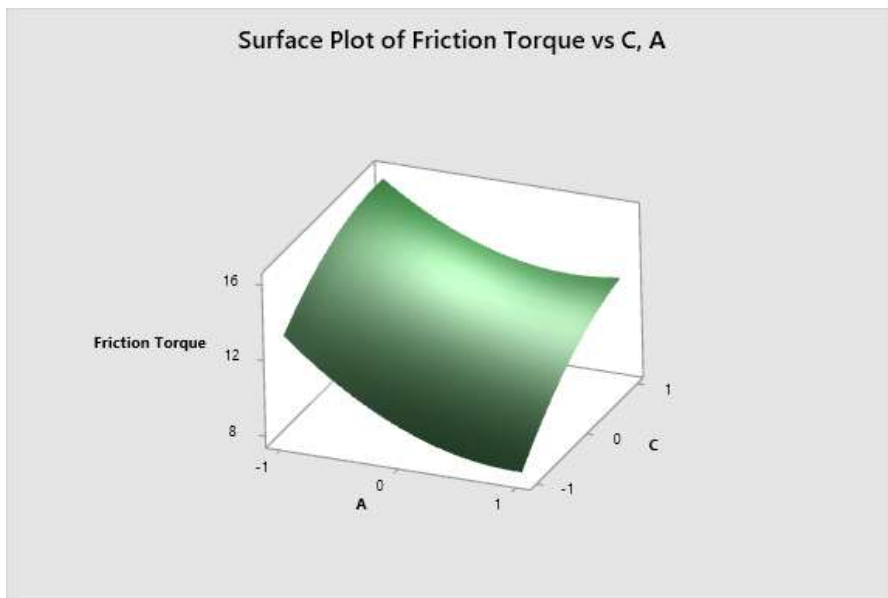
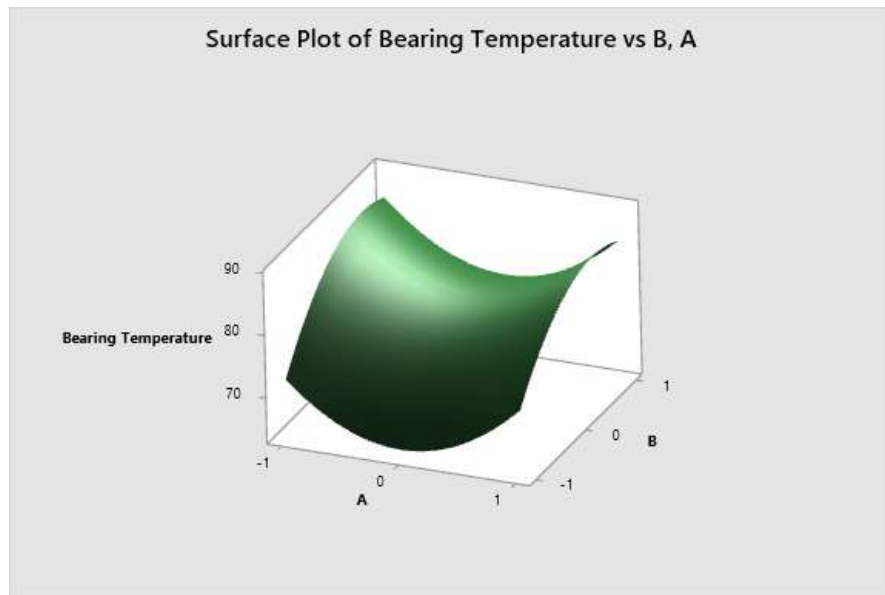
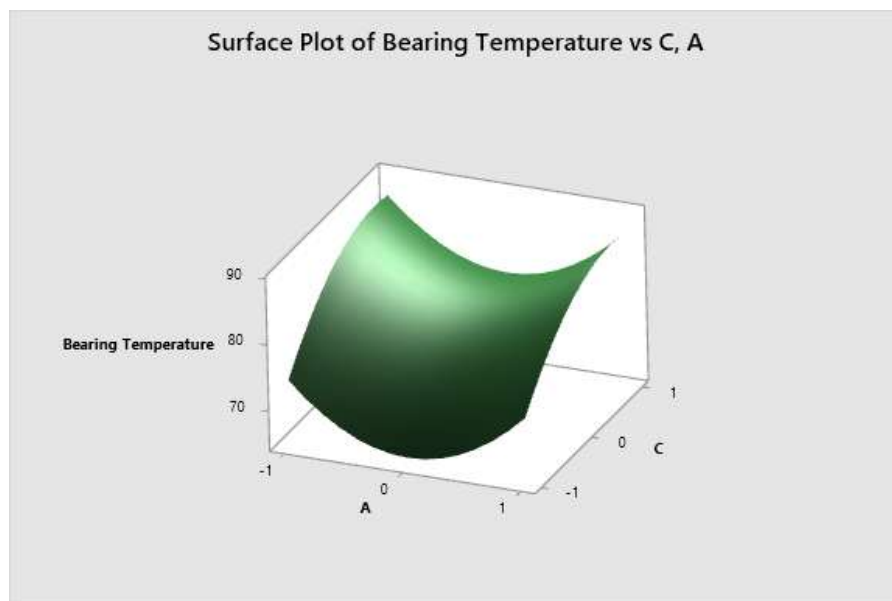


FIGURE 4.25 Surface Plot of FT vs C, A

**FIGURE 4.26 Surface Plot of BT vs B, A****FIGURE 4.27 Surface Plot of BT vs C, A**

Overall, the contour and surface plot analyses reveal that rib convexity has a significant influence on both FT and BT. While increasing rib convexity effectively reduces FT, an optimum convexity of approximately $3\text{ }\mu\text{m}$ is required to minimize BT. The results further indicate that lower axial loads and rotational speeds are favorable for reducing both response variables, thereby contributing to improved bearing performance and operating efficiency.

4.7 Comparison of FFD and RSM BBD Models

To evaluate the effectiveness of the developed predictive models, a comparison was made between the FFD and RSM BBD models. The comparison was performed using statistical indicators such as the coefficient of determination (R^2), adjusted coefficient of determination (Adjusted R^2), and predicted coefficient of determination (Predicted R^2) for both FT and BT responses.

The comparison results as shown in TABLE 4.13 indicate that both modeling approaches provide excellent agreement with the experimental data, as evidenced by the high values of R^2 , Adjusted R^2 , and Predicted R^2 . For FT, the FFD model achieved R^2 , Adjusted R^2 , and Predicted R^2 values of 98.87%, 98.27%, and 97.10%, respectively. In comparison, the RSM Box–Behnken model exhibited improved values of 99.74%, 99.27%, and 95.85%, respectively. The higher R^2 and Adjusted R^2 values indicate that the Box–Behnken model provides a better representation of the experimental response.

TABLE 4.13 Comparison of Statistical Parameters of FFD and RSM BBD Models

Criteria	FFD		RSM BBD	
Number of Runs	27 for 3 factors, 3 levels		15 for 3 factors, 3 levels	
Design Structure	Tests all combinations		Midpoints of edges + centre	
Model Fit (R^2 , Adjusted R^2 , Predicted R^2)	FT	BT	FT	BT
	98.87%	99.36%	99.74%	99.21%
	98.27%	99.02%	99.27%	97.79%
	97.10%	98.16%	95.85%	96.75%
Adequacy of Model (p-values, Lack of Fit)	Report F-test, p-values most p-values are < 0.05		Report F-test, p-values most p-values are < 0.05	
Error between predicted and actual (Max % error)	FT and BT shows 5.56% and 2.49% respectively		FT and BT shows 2.08% and 2.7%, respectively	
Efficiency	High number of runs		Fewer runs	
Interaction Detection	Very good		Moderate (some corners not tested)	

Similarly, for BT, the FFD model produced R^2 , Adjusted R^2 , and Predicted R^2 values of 99.36%, 99.02%, and 98.16%, respectively. The corresponding values obtained using the BBD were 99.21%, 97.79%, and 96.75%, respectively. Although both models demonstrate excellent predictive capability, the statistical indicators reveal only marginal differences between the two approaches.

Overall, the results confirm that both FFD and RSM BBD are capable of accurately modeling FT and BT within the selected experimental domain. However, the BBD requires fewer experimental runs while still maintaining a high level of accuracy and model adequacy. Therefore, the RSM BBD offers a more efficient and economical approach for modeling and optimization of TRB performance characteristics.

The comparison further demonstrates that the developed Box–Behnken models are sufficiently accurate for response prediction and optimization, thereby justifying their selection for subsequent MRO using DFA and Grey Relational Analysis (GRA).

4.8 Conclusions of Comparison between FFD and RSM BBD Models

The comparison between the FFD and RSM BBD models demonstrates that both approaches are highly effective in modeling the FT and BT responses of TRBs. The following conclusions can be drawn:

- Both modeling approaches exhibited coefficient of determination (R^2) values greater than 95%, indicating excellent agreement between the experimental and predicted results.
- The statistical indicators, including R^2 , Adjusted R^2 , and Predicted R^2 , confirm that both Full Factorial and Box–Behnken models are statistically adequate and capable of accurately representing the behaviour of the selected response variables.
- The BBD provides comparable predictive accuracy while requiring fewer experimental runs, resulting in reduced experimentation time, cost, and resource utilization.
- Due to its high predictive capability and experimental efficiency, the BBD is more suitable for response prediction, process optimization, and engineering applications where minimizing experimental effort is important.
- The FFD remains advantageous when a comprehensive investigation of all possible factor combinations and interaction effects is required, particularly during the initial stages of process exploration.

- For the present TRB study, the BBD offers a practical and reliable alternative to the FFD, providing accurate response modeling with a lower experimental burden.

Therefore, while both methods are statistically sound and validated, the BBD is preferred for optimization studies due to its efficiency and strong predictive performance, whereas the FFD is more appropriate when exhaustive analysis of factor interactions is desired.

4.9 Conclusion

The experimental data were systematically analysed using FFD and RSM BBD. The L27 FFD established the effects of rib convexity, axial load, and operating speed on FT and BT, with predicted R^2 values of 97.10% for FT and 98.16% for BT. The RSM BBD models provided predicted R^2 of 95.85% for FT and 96.75% for BT. The results showed that FT generally increased with axial load and speed and decreased with rib convexity, whereas BT exhibited dependence on rib convexity, with favourable thermal behaviour near 3 μm rib convexity. Comparison of the modelling approaches showed that BBD required only 15 experimental runs compared with 27 runs for FFD, while maintaining comparable predictive capability.

CHAPTER 5

Results and Discussion

5.1 Introduction

This chapter presents the optimization of TRB parameters using four different simultaneous optimization methods. The objective of the optimization is to determine the optimal combination of rib convexity, axial load, and rotational speed that simultaneously optimise FT and BT. The optimization results obtained from both techniques are compared and validated through confirmation experiments. The effectiveness of the developed models and optimization approaches is also discussed in relation to bearing performance.

5.2 Simultaneous Optimization Methods

Optimization plays a vital role in engineering design and manufacturing processes, particularly when multiple performance characteristics must be improved simultaneously. In TRB applications, minimizing FT and BT is essential for enhancing efficiency, reducing energy losses, and extending bearing service life. Since these responses are influenced by several interacting process parameters, a systematic optimization approach is required to identify the most favourable operating conditions.

Various statistical and MRO techniques have been developed to address such problems. The selection of an appropriate optimization method depends on factors such as the complexity of the system, the number of response variables, the availability of experimental data, and the desired accuracy of prediction (Ajith Arul Daniel et al., 2019; Passari et al., 2022). In the present study, FFD, RSM BBD, Desirability Function Analysis (DFA), and Grey Relational Analysis (GRA) were employed to investigate and optimize the performance of TRBs.

In the present investigation, the predictive capabilities of FFD and RSM BBD are first utilized to model the bearing responses using MINITAB 19. Subsequently, DFA and GRA are employed for simultaneous optimization of FT and BT. The optimization results obtained

from these techniques are compared to identify the most effective approach for improving TRBs performance.

5.3 FFD Optimisation using MINITAB 19

The FFD approach provides a comprehensive evaluation of all possible combinations of factor levels and enables the identification of main effects and interaction effects with high accuracy. The optimization of TRB operating parameters was initially carried out using the Response Optimizer tool available in MINITAB 19 based on the FFD experimental results. The objective of the optimization was to determine the combination of rib convexity (A), axial load (B), and rotational speed (C) that simultaneously minimizes FT and BT.

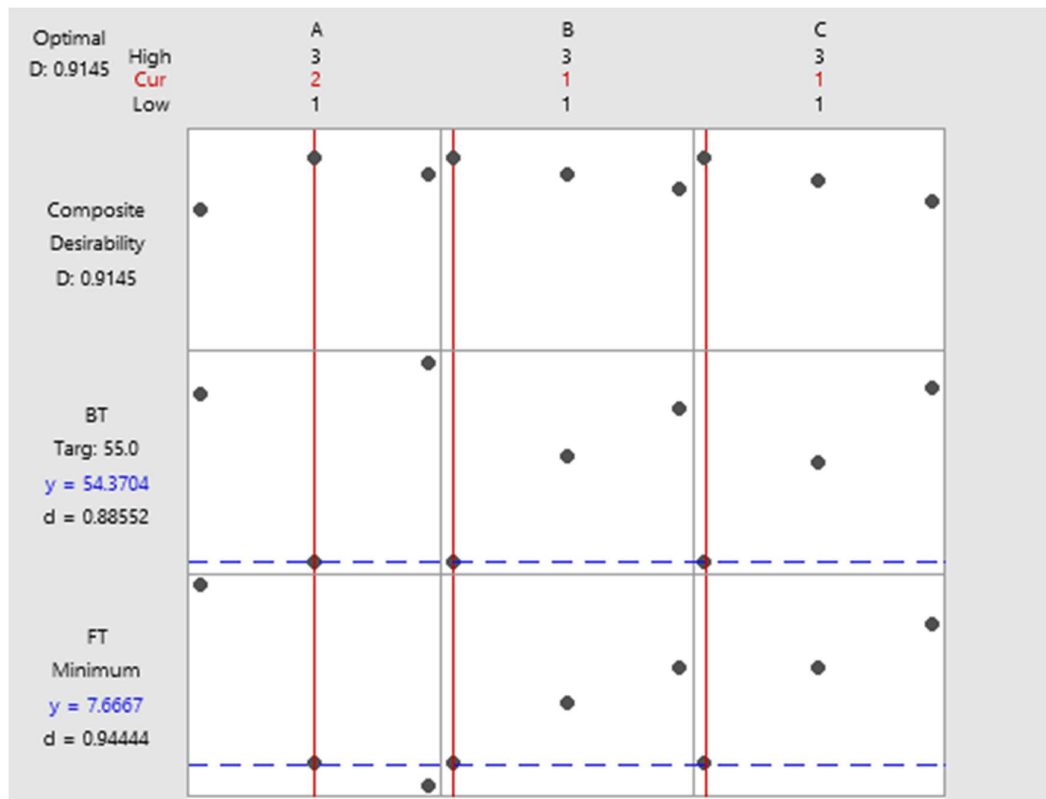


FIGURE 5.1 Graphical Optimization of FFD Optimization Using MINITAB 19

FIGURE 5.1 presents the graphical optimization results generated by MINITAB 19. The response optimizer evaluates the influence of the selected process parameters on the response variables and identifies the factor settings that maximize the overall desirability. The desirability function combines multiple responses into a single performance index, where a desirability value closer to one indicates a more favourable operating condition. The

optimum parameter combination obtained from the optimization study is presented in TABLE 5.1, which shows the first three optimal solutions in tabular form. Among the available solutions, the best optimum condition corresponds to factor levels A₂, B₁, and C₁, with a composite desirability value of 0.9145. This result indicates that the selected combination of rib convexity, axial load, and rotational speed provide the most desirable balance between minimizing FT and normalizing BT within the investigated experimental domain.

TABLE 5.1 FFD Optimization Solution Obtained by MINITAB 19

Solutions	A	B	C	BT Fit	FT Fit	Composite Desirability
1	2	1	1	54.3704	7.66667	0.914509
2	3	1	1	67.3704	7.11111	0.831756
3	2	2	1	61.2593	9.22222	0.830916

5.4 Desirability Function Analysis (DFA)

DFA is one of the most widely used techniques for MRO. The method transforms individual response variables into dimensionless desirability values and combines them into a single composite desirability index. This approach allows multiple responses to be optimized simultaneously while considering the relative importance of each response. Due to its flexibility and ease of implementation, DFA has been extensively adopted in industrial optimization applications. DFA is popular technique first introduced by (Fisher, 1974) and popularised by (Derringer, G. and Suich, 1980) considered it as useful approach to optimize multiple responses.

According to DFA, first each response converted into individual desirability index d_i that varies between range (0, 1). When the response reaches the goal or target then $d_i = 1$ and when it fails to reach or not within acceptable range then $d_i = 0$. Desirability index calculated according desirability functions (Montgomery, 2013) Nominal-the-best Equation 5.1, Lager-the-better Equation 5.2 and Smaller-the-best Equation 5.3 (Montgomery, 2010). In these equations y is response value, L is minimum of the responses, U is maximum of the responses and T is targeted value for the response. Here r , r_1 and r_2 indicated weightage for

each response normally taken as 1. Here both responses FT and BT are considered important performance indicators in the present investigation as FT represents the mechanical power loss associated with bearing operation, whereas BT represents the resulting thermal behaviour and operating condition of the bearing. Since the objective of the study was to achieve a balanced improvement in both responses without assigning preferential importance to either response an equal weight of 1 was assigned to each response in the desirability analysis. Then rank based design factors are selected as given by Equation 5.4 to maximize (optimize) the overall desirability D. Here Smaller-the-better type is considered for the FT and Nominal-the-best is considered for the BT. Targeted value for the FT is considered as minimum from the responses and targeted value for BT considered as 65° based on the stable operating behaviour observed during the pilot experiments. At approximately 400 rpm, the bearing exhibited stable operation with a maximum temperature of approximately 66°C at the selected upper load condition, without abnormal wear or significant surface damage. Therefore, 65°C was adopted as a representative desirable operating temperature and it is preferable bearing operating temperature according to (SKF, 2003).

i) Nominal-the-best: if target T is considered between the Lower L and Upper U limits for the response y.

$$di = \begin{cases} 0 & y < L \\ \left(\frac{y-L}{T-L}\right)^{r1} & L \leq y \leq T \\ \left(\frac{U-y}{U-T}\right)^{r2} & T \leq y \leq U \\ 1 & y > U \end{cases} \quad 5.1$$

ii) Larger-the-better: if target T is the maximum value for the response y.

$$di = \begin{cases} 0 & y < L \\ \left(\frac{y-L}{T-L}\right)^r & L \leq y \leq T \\ 1 & y > T \end{cases} \quad 5.2$$

iii) Smaller-the-better: if target T is the minimum value for the response y.

$$di = \begin{cases} 0 & y < T \\ \left(\frac{y-L}{T-L}\right)^r & T \leq y \leq U \\ 1 & y > U \end{cases} \quad 5.3$$

$$D = (d1.d2.d3 \dots dm)^{1/m} \quad 5.4$$

Where D is overall desirability of the responses, m represents number of responses. TABLE 5.2 shows the calculated desirability index and overall desirability (D) for each response along with rank using DFA. Using the experimental results the individual desirability values and the overall desirability index were calculated for each experimental trial. The ranking results indicate that the highest overall desirability was obtained at factor combination A₂B₁C₁, corresponding to a rib convexity of 3 µm, axial load of 2000 kgf, and rotational speed of 400 rpm.

TABLE 5.2 Overall Desirability Calculation Using DFA Optimization

Ex. No	Factors			Experiment Value		Desirability Index		Overall Desirability (D)	Rank
	A	B	C	FT (Nm)	BT (°C)	FT	BT		
1	1	1	1	11.8611	66.2315	0.76899	0.83356	0.89478	9
2	1	1	2	13.7500	72.0926	0.65413	0.74235	0.83477	16
3	1	1	3	14.6389	78.7315	0.59242	0.62312	0.77947	19
4	1	2	1	13.2222	73.4259	0.68816	0.71999	0.83898	13
5	1	2	2	15.4444	80.037	0.53033	0.59688	0.75008	22
6	1	2	3	16.6667	87.4259	0.41890	0.41841	0.64704	24
7	1	3	1	14.5833	79.0648	0.59646	0.61653	0.77873	20
8	1	3	2	17.1389	86.4259	0.36689	0.44676	0.63629	25
9	1	3	3	18.6944	94.5648	0.00000	0.00000	0.00000	27
10	2	1	1	8.0000	53.787	0.96202	1.00000	0.99037	1
11	2	1	2	10.2222	59.4815	0.85626	0.92755	0.94403	4
12	2	1	3	11.4444	65.9537	0.79209	0.83764	0.90252	7
13	2	2	1	9.1111	60.1481	0.91067	0.91870	0.95639	2
14	2	2	2	11.6667	66.5926	0.77985	0.82823	0.89648	8
15	2	2	3	13.2222	73.8148	0.68816	0.71334	0.83704	15
16	2	3	1	10.2222	64.9537	0.85626	0.85215	0.92423	5
17	2	3	2	13.1111	72.1481	0.69511	0.74144	0.84729	12
18	2	3	3	15.0000	80.1204	0.56543	0.59517	0.76165	21
19	3	1	1	7.1389	68.1204	1.00000	0.80529	0.94730	3

20	3	1	2	9.6944	73.6481	0.88252	0.71620	0.89164	10
21	3	1	3	11.2500	79.9537	0.80264	0.59859	0.83255	17
22	3	2	1	8.0000	73.6481	0.96202	0.71620	0.91108	6
23	3	2	2	10.8889	79.9259	0.82188	0.59916	0.83770	14
24	3	2	3	12.7778	86.9815	0.71555	0.43124	0.74531	23
25	3	3	1	8.8611	77.6204	0.92248	0.64462	0.87814	11
26	3	3	2	12.0833	84.6481	0.75638	0.49314	0.78150	18
27	3	3	3	14.3056	92.4537	0.61628	0.22753	0.61193	26

The optimization results confirm that this parameter combination provides the most desirable compromise between minimizing FT and maintaining the BT near its preferred operating condition. The graphical representation of the optimization results obtained using MINITAB 19 is presented in FIGURE 5.2. The highest desirability value was achieved at the rank-one condition, indicating that the selected parameter combination is the optimum operating condition within the investigated experimental range.

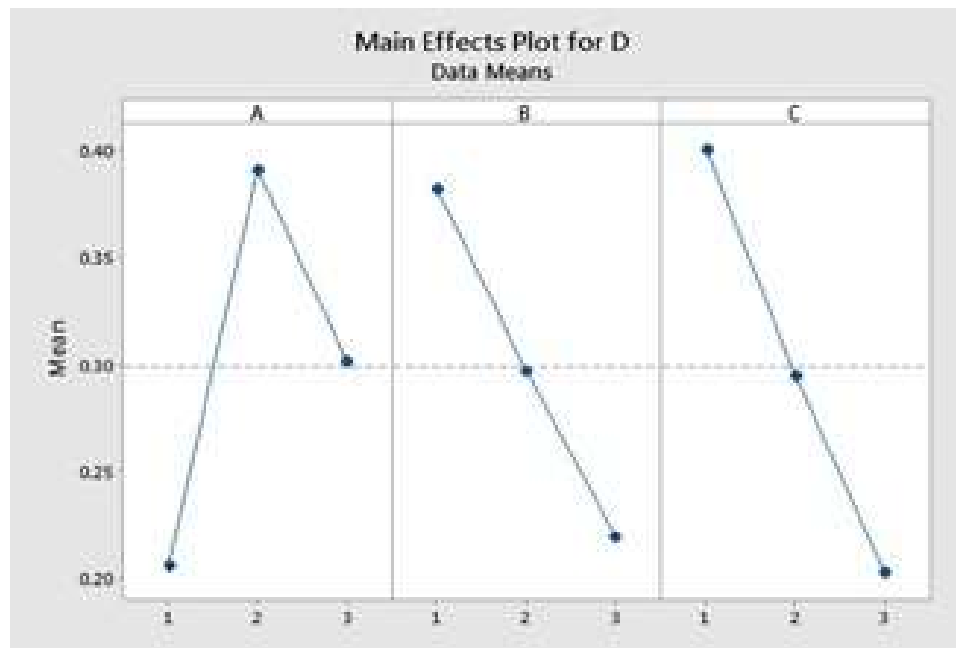


FIGURE 5.2 Graphical Representation DFA Optimization

5.5 RSM based BBD optimization using Minitab 19

RSM based BBD (RSM BBD) is an efficient statistical technique used to develop predictive models and optimize process parameters. By requiring fewer experimental runs than a FFD, it effectively models nonlinear relationships among variables while maintaining high predictive accuracy. The developed RSMs can subsequently be used for process optimization and response prediction within the selected experimental region (Yıldız et al., 2023).

RSM BBD was employed to determine the optimum combination of process parameters and performed using the response optimizer tool available in MINITAB 19. It utilizes the developed regression models to predict the response values and identify the parameter settings that maximize the overall desirability.

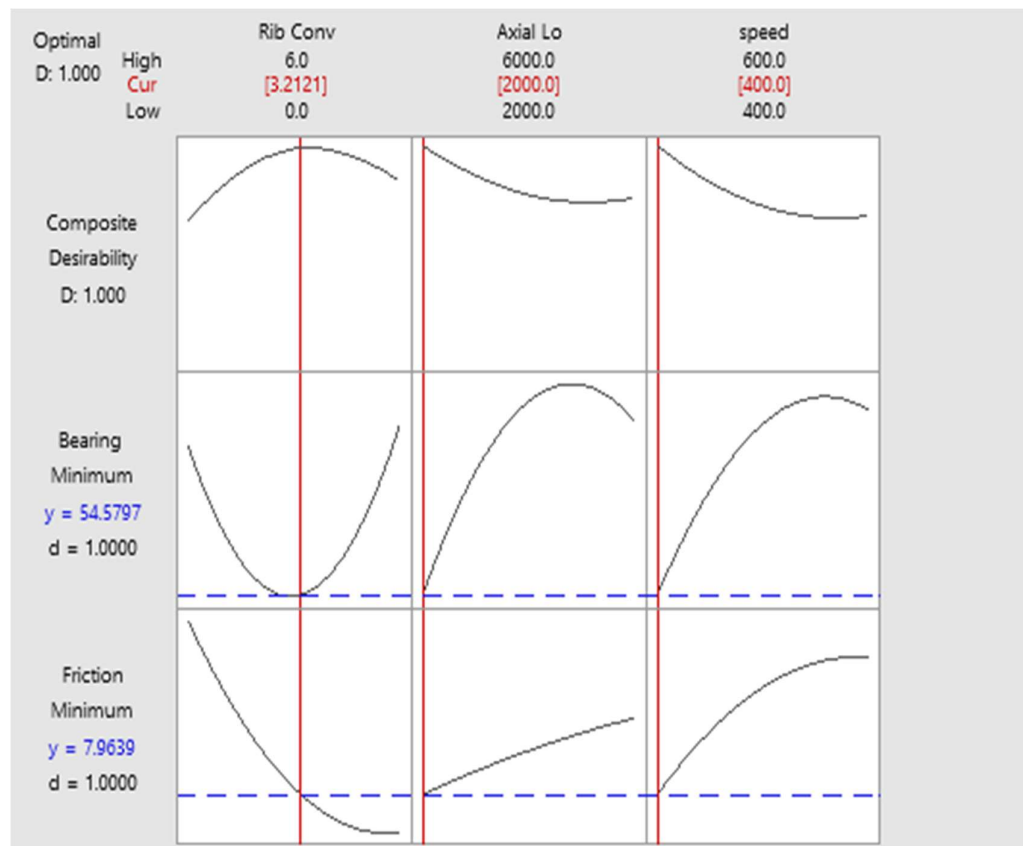


FIGURE 5.3 Graphical Representation of DFA Optimization Using MINITAB 19

TABLE 5.3 BBD RSM Optimization Solution Obtained by MINITAB 19

Solutions	Rib Convexity	Axial Load	speed	Bearing Temperature Fit	Friction Torque Fit	Composite Desirability
1	3.21212	2000.00	400	54.5797	7.9639	1.00000
2	4.35654	6000.00	400	65.8386	8.8561	0.77356
3	0.65605	2000.00	400	59.5271	10.8200	0.76780

The optimization process simultaneously considers the effects of rib convexity (A), axial load (B), and rotational speed (C) on the selected response variables. The objective was to obtain the most favourable operating condition that minimizes FT while maintaining the BT within the desired operating range. The optimization criterion was established by assigning appropriate response goals and desirability functions to the individual responses.

FIGURE 5.3 presents the graphical optimization results generated by the Response Optimizer in MINITAB 19. The graphical representation illustrates the influence of the process parameters on the predicted responses and identifies the optimum factor settings corresponding to the maximum overall desirability.

TABLE 5.3 shows three best solutions in tabulated optimization results, including the optimum parameter combination, predicted response values, and composite desirability. The optimization results indicate that the optimum combination of process parameters is at $A_2B_1C_1$ to minimize FT and BT. The predicted response values obtained from the developed RSM models demonstrate the effectiveness of the optimization process and confirm the adequacy of the regression equations developed in Chapter 4. Compared with the FFD optimization, the RSM BBD approach provides a more refined optimization result by utilizing second-order regression models and response surface analysis. The method enables continuous prediction of response values within the experimental domain and facilitates the identification of optimum operating conditions without requiring exhaustive experimentation.

5.6 RSM BBD based Grey Relational Analysis

5.6.1 Grey Relational Analysis (GRA)

GRA is another powerful multi objective optimization technique particularly suitable for systems involving limited, uncertain, or complex data to transform the multi objective problems in single normalized objective. Unlike conventional statistical optimization methods, GRA does not require the development of mathematical regression models and can effectively handle nonlinear relationships among process parameters and responses. The main advantages of this method is that it can equally effective with qualitative and quantitative data. This method is relatively simple and gives reasonably expected output with minimal data requirement. The method converts multiple performance characteristics into a single GRG, which is then used to determine the optimal parameter combination. (Ajith Arul Daniel et al., 2019; Gopalakrishnan et al., 2021).

Step 1: Normalization

If the original sequence is targeted for maximize then it is normalized as per 'the-larger-the-better' and the calculation is according to Equation 5.5,

$$X^*_i(k) = \frac{X^o_i(k) - \min X^o_i(k)}{\max X^o_i(k) - \min X^o_i(k)} \quad 5.5$$

If it is 'the-smaller-the-better', then the calculation is according to Equation 5.6,

$$X^*_i(k) = \frac{\max X^o_i(k) - X^o_i(k)}{\max X^o_i(k) - \min X^o_i(k)} \quad 5.6$$

And if the case where there is 'a specific value' for target the calculation is according to Equation 5.7,

$$X^*_i(k) = 1 - \frac{|X^o_i(k) - T|}{\max\{\max X^o_i(k) - T, T - \min X^o_i(k)\}} \quad 5.7$$

Where $X_o(k)$ and $X_i(k)$, $i = 1, 2, 3 \dots m$; $k = 1, 2, 3 \dots n$, are original reference sequence and comparison sequence respectively. m is total number of experiment and n is total number of observations. T is targeted value.

Step 2: Grey Relational Coefficient (GRC)

Next step is to identify GRC, can be achieved by Equation 5.8,

$$\gamma(X_o^*(k), X_j^*(k)) = \frac{\Delta \min - \beta \Delta \max}{\Delta_{oi}(k) - \beta \Delta \max} \quad 5.8$$

Where β is distinguishing coefficient ranges (0, 1) and $\Delta_{oi}(k)$, $\Delta \min$ and $\Delta \max$ are the deviation of the reference $X_o^*(k)$ and comparability reference $X_i^*(k)$, that is calculated using Equation 5.9

$$\begin{aligned} \Delta_{oi}(k) &= |X_o^*(k) - X_i^*(k)| \\ \Delta \min &= \min |X_o^*(k) - X_j^*(k)| \\ \Delta \max &= \max |X_o^*(k) - X_j^*(k)| \end{aligned} \quad 5.9$$

Step 3: Grey Relational Grade (GRG)

A GRG is the weighted sum of GRC and can be calculated as shown in Equation 5.10,

$$\begin{aligned} \text{GRG}, \gamma(X_o^*, X_i^*) &= \sum_{k=1}^n \beta_k \gamma(X_o^*(k), X_i^*(k)) \\ \sum_{k=1}^n \beta_k &= 1 \end{aligned} \quad 5.10$$

Here GRG, $\gamma(X_o^*, X_i^*)$ presents the correlation level between comparability and reference sequence. If a specific comparability sequence has more importance than any reference sequence, the GRG for such comparability sequence and the reference sequence will exceed than other GRG.

GRA Calculation Procedure TABLE 5.4 presents the detailed calculations involved in the GRA procedure. The GRA values were determined systematically by applying the equations

discussed in the previous section. The analysis was performed using the response values predicted by the RSM regression models developed in Chapter 4 for FT and BT.

TABLE 5.4 GRG Calculation Using GRA Optimization

Factors and Its Levels			Experimental Results		RSM Regression Model		Normalization		Deviation Sequence		Grey Relational Coefficient		GRG	Rank
A	B	C	FT	BT	FT FITS	BT FITS	FT	BT	FT	BT	FT	BT		
-1	-1	0	14	72	13.75	72.375	0.3562	0.4479	0.6438	0.5521	0.4371	0.4752	0.4562	10
1	-1	0	10	73	10	73.375	0.7671	0.4170	0.2329	0.5830	0.6822	0.4617	0.5720	5
-1	1	0	17	86	17	85.625	0.0000	0.0386	1.0000	0.9614	0.3333	0.3421	0.3377	15
1	1	0	12	85	12.25	84.625	0.5205	0.0695	0.4795	0.9305	0.5105	0.3495	0.4300	11
-1	0	-1	13	74	13.125	74.125	0.4247	0.3938	0.5753	0.6062	0.4650	0.4520	0.4585	9
1	0	-1	8	74	7.875	74.125	1.0000	0.3938	0.0000	0.6062	1.0000	0.4520	0.7260	2
-1	0	1	16	87	16.125	86.875	0.0959	0.0000	0.9041	1.0000	0.3561	0.3333	0.3447	14
1	0	1	13	87	12.875	86.875	0.4521	0.0000	0.5479	1.0000	0.4771	0.3333	0.4052	12
0	-1	-1	8	55	8.125	54.5	0.9726	1.0000	0.0274	0.0000	0.9481	1.0000	0.97403	1
0	1	-1	10	64	9.875	64.25	0.7808	0.6988	0.2192	0.3012	0.6952	0.6241	0.6597	3
0	-1	1	11	65	11.125	64.75	0.6438	0.6834	0.3562	0.3166	0.5840	0.6123	0.5981	4
0	1	1	15	79	14.875	79.5	0.2329	0.2278	0.7671	0.7722	0.3946	0.3930	0.3938	13
0	0	0	12	78	12	76	0.5479	0.3359	0.4521	0.6641	0.5252	0.4295	0.4773	6
0	0	0	12	74	12	76	0.5479	0.3359	0.4521	0.6641	0.5252	0.4295	0.4773	6
0	0	0	12	76	12	76	0.5479	0.3359	0.4521	0.6641	0.5252	0.4295	0.4773	6

The GRA procedure consists of several stages, including normalization of the response values, determination of the deviation sequences, calculation of GRC, and finally the computation of GRG. Since FT and BT have different units and ranges, normalization was first carried out to transform the responses into a comparable dimensionless form. The deviation sequences were then obtained by calculating the absolute difference between the normalized values and the ideal reference sequence. Subsequently, the GRCs were evaluated to express the relationship between each experimental run and the ideal condition.

Finally, the GRG was calculated as the average of the GRCs of FT and BT, representing the overall performance index for MRO.

As shown in TABLE 5.4 the maximum GRG value of 0.9740 was obtained for the parameter combination $A = 0$, $B = -1$, and $C = -1$ (Run 9). According to the GRA principle, a higher GRG indicates a closer relationship with the ideal performance characteristics and therefore represents a better overall response combination. Hence, this experimental condition can be considered as the optimum parameter setting among all the investigated runs for simultaneously optimizing FT and BT.

5.7 Comparison of Optimization Techniques

TABLE 5.5 presents a comparative evaluation of the four optimization techniques employed in the present study, namely FFD optimization using MINITAB 19, DFA, RSM BBD optimization using MINITAB 19, and RSM BBD based GRA. The comparison was carried out based on the optimized process parameters, predicted response values, and the corresponding composite desirability or GRG.

TABLE 5.5 Comparison of Optimization Techniques

Method	A	B	C	BT Fit	FT Fit	Optimization Matric
Full Factorial Optimization (Using MINITAB 19)	3	2000	400	54.3704	7.6667	Composite Desirability = 0.91451
DFA	3	2000	400	53.787	8.000	Composite Desirability = 0.99037
RSM-BBD Optimization (Using MINITAB 19)	3.2121	2000	400	54.5797	7.9639	Composite Desirability = 1.00000
RSM BBD based GRA	3	2000	400	54.5	8.125	GRG = 0.97403

The FFD optimization and DFA both identified the same optimal factor combination corresponding to $A_2B_1C_1$, indicating that a rib convexity of 3 μm , axial load of 2000 kgf, and speed of 400 rpm provide favourable operating conditions for the bearing system. However, DFA produced a significantly higher composite desirability value of 0.99037 compared to 0.91451 obtained from the direct Full Factorial optimization.

The RSM BBD optimization technique generated the highest overall composite desirability value of 1.00000. The optimum parameter combination predicted by the RSM BBD model was a rib convexity of 3.2121 μm , axial load of 2000 kgf, and speed of 400 rpm. At these conditions, the predicted BT and FT were 54.5797 $^{\circ}\text{C}$ and 7.9639 Nm, respectively. The desirability value of unity indicates that the obtained solution completely satisfied the optimization objectives within the investigated design space. Therefore, RSM BBD optimization was identified as the most effective technique for simultaneous optimization of FT and BT.

The RSM BBD based GRA approach with a GRG of 0.97403. The optimum factor combination obtained through GRA corresponded to the coded levels $A = 0$, $B = -1$, and $C = -1$. The predicted BT and FT at this condition were 54.5 $^{\circ}\text{C}$ and 8.125 Nm, respectively.

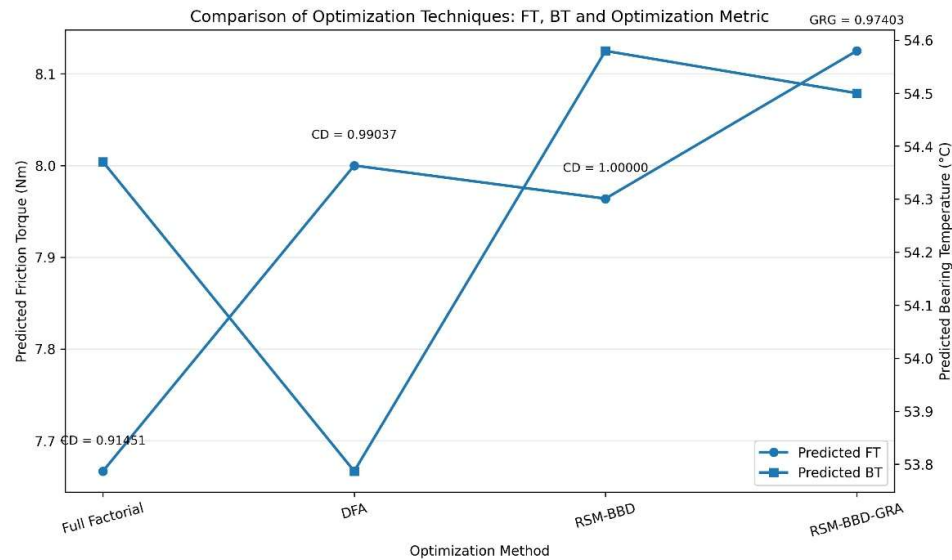


FIGURE 5.4 Comparative analysis of predicted FT and BT for different optimization techniques

FIGURE 5.4 compares the predicted FT and BT obtained using different optimization techniques, along with their respective optimization metrics. The results show close

agreement among the methods, with all methods identifying an axial load of 2000 kgf and a speed of 400 rpm, while the optimum rib convexity is approximately 3 μm .

Among the investigated methods, RSM BBD optimization using MINITAB 19 provided the most favourable optimization solution within the desirability-based framework, with a composite desirability of 1.000 and predicted responses of 7.96 Nm FT and 54.57°C BT. DFA and Full Factorial optimization also identified operating conditions close to 3 μm rib convexity, 2000 kgf axial load, and 400 rpm. The RSM BBD based GRA approach identified a comparable optimum condition, demonstrating the consistency of the optimization results. Since composite desirability and GRG are method specific indices, their numerical values were not directly compared against each other.

5.8 Confirmation Experiments

Confirmation experiments are conducted after the development of predictive models and optimization of process parameters to verify the accuracy and reliability of the obtained results. The primary purpose of a confirmation experiment is to validate whether the optimum operating conditions predicted by statistical models and optimization techniques can be reproduced under actual experimental conditions. It serves as the final verification step to ensure that the developed models accurately represent the real behaviour of the system.

5.8.1 Confirmation Experiments for FFD Optimization

Confirmation experiments were conducted to validate the optimum parameter combinations obtained from the FFD optimization. The predicted response values generated from the optimization model were compared with the experimentally observed values under the same process conditions. The purpose of the confirmation experiments was to verify the adequacy and reliability of this developed model and to evaluate the accuracy of the predicted responses. TABLE 5.6 presents the comparison between the predicted and experimental values of FT and BT for the three optimum parameter combinations obtained through FFD optimization.

TABLE 5.6 Confirmation Experiments Based on Optimal Parameters Obtained Using FFD

FFD Optimum Solutions	A (μ)	B (kgf)	C (rpm)	Status	FT (Nm)	BT ($^{\circ}$ C)
1	2.8	2000	400	Predicted	7.66	54.37
				Experimental	8	56
				Deviation %	4.25	2.91
2	6.1	2000	400	Predicted	7.11	67.37
				Experimental	7	65
				Deviation %	1.57	3.65
3	3	4000	400	Predicted	9.22	61.25
				Experimental	9	63
				Deviation %	2.44	2.78

The results indicate that the experimental values are in close agreement with the predicted values for all three optimum solutions. The maximum deviation observed for FT was 4.25%, while for BT it was 3.65%. Such small deviations confirm the validity of the optimization model and demonstrate that the predicted optimum conditions can reliably achieve the desired process performance. Therefore, the FFD optimization approach can be considered effective for predicting and optimizing the process parameters within the investigated range.

5.8.2 Confirmation Experiments for RSM BBD Optimization

TABLE 5.7 presents the comparison of predicted and experimental values of FT and BT at the optimum parameter setting obtained through RSM BBD optimization. The confirmation experiment was carried out using the optimized levels of factors A, B, and C, and the resulting responses were recorded. To validate the optimum parameter combination obtained through RSM optimization, again second confirmation experiments were performed under the predicted optimum process conditions. Similar to the FFD validation procedure, the experimental results were compared with the response values predicted by the RSM model. The maximum and minimum percentage deviation for FT are 1.64 % and 0.45 % respectively between the predicted and experimental responses. The close agreement between the predicted and experimental values demonstrates the adequacy of the developed RSM model and confirms the effectiveness of the optimization procedure.

TABLE 5.7 Confirmation Experiments Based on Optimal Parameters Obtained Using RSM

RSM Optimum Solutions	A (μ)	B (kgf)	C (rpm)	Status	FT (Nm)	BT ($^{\circ}$ C)
1	3.2	2000	400	Predicted	7.96	54.57
				Experimental	8	56
				Deviation %	0.45	2.54
2	4.3	6000	400	Predicted	8.86	65.84
				Experimental	9	68
				Deviation %	1.60	3.18
3	0.6	2000	400	Predicted	10.82	59.53
				Experimental	11	62
				Deviation %	1.64	3.99

The deviation percentages obtained from the confirmation experiments were found to be within acceptable limits for both the experiments, indicating that the developed FFD and RSM models can accurately predict the process responses. Hence, the optimized parameter combination determined by FFD and RSM can be considered reliable for achieving the desired performance characteristics.

5.9 Conclusion

The multi response optimization study demonstrated that the investigated optimization techniques consistently identified an operating condition close to 3 μ m rib convexity, 2000 kgf axial load, and 400 rpm. The RSM BBD optimization using MINITAB 19 provided the most favourable solution, with an optimum rib convexity of 3.2121 μ m, predicted FT of 7.9639 Nm, BT of 54.5797 $^{\circ}$ C, and composite desirability of 1.000. FFD and DFA optimization also identified the same practical factor combination of approximately 3 μ m, 2000 kgf, and 400 rpm with composite desirability of 0.91451 and 0.99037 respectively. Optimization using GRA method also identified a comparable optimum condition with GRG of 0.97403. Confirmation experiments verified the predictive capability of the models, with RSM based FT deviations between 0.45% and 1.64%, and BT deviations between 2.54%

and 3.99%. Thus, the developed optimization framework provides reliable operating conditions for simultaneous control of FT and BT within the investigated range.

CHAPTER 6

Conclusions and Scope of Future Work

6.1 Conclusion

The present experimental investigation on TRBs provides a comprehensive understanding of the influence of inner race rib convexity, axial load, and operating speed on FT and BT. The developed experimental methodology, combined with statistical modelling and multi-response optimization techniques, establishes a systematic approach for analysing the tribological and thermal behaviour of TRBs under high axial loading conditions. Furthermore, the comparative application of FFD, RSM BBD, DFA, and GRA demonstrates an effective framework for developing predictive models and identifying optimum operating conditions for improved bearing performance.

Key Findings:

- Experimental results showed that FT increased with increasing axial load and operating speed, whereas it decreased with increasing rib convexity. In contrast, BT was minimum at approximately 3 μm rib convexity, indicating an optimum balance between friction reduction and contact stress.
- The FFD L27 successfully modelled the bearing responses, and the developed regression models achieved prediction accuracies of 98.87% and 99.36% for FT and BT respectively.
- The RSM BBD developed highly accurate predictive models with R^2 values greater than 99%, while requiring fewer experimental runs than the Full Factorial Design.
- The ANOVA results indicate that the statistical significance of the factors varies depending on the response and the model. For the FFD friction-torque model, rib convexity and operating speed are statistically significant, whereas axial load is statistically insignificant at the 95% confidence level. For the BBD bearing-temperature model, axial load and operating speed are significant, while the linear effect of rib convexity is insignificant; however, its quadratic term is significant, indicating a nonlinear effect of rib convexity on bearing temperature.

- A comparative study between FFD and RSM BBD demonstrated that both approaches produced statistically reliable models; however, the RSM BBD required fewer experimental runs while maintaining comparable prediction accuracy, making it more suitable for experimental optimization.
- Multi-response optimization using Full Factorial Design, DFA, RSM-BBD, and GRA consistently identified the optimum operating condition close to 3 μm rib convexity, 2000 kgf axial load, and 400 rpm.
- Among the investigated optimization techniques, RSM-BBD provided the best overall performance, predicting an optimum condition at 3.21 μm rib convexity, 2000 kgf axial load, and 400 rpm, with a minimum FT of 7.96 Nm and BT of 54.57°C.

6.2 Chapter wise Conclusions

Chapter 1 – Introduction

This chapter presented the background and significance of rolling contact bearings, including their history, market trends, components, and classification. It introduced tapered roller bearings (TRBs), their applications, research problem, components, geometry, numbering system, and characteristics. The discussion established the technical foundation and motivation for the experimental investigation of friction torque in TRBs presented in the subsequent chapters.

Chapter 2 – Literature Review

The literature review established that FT is a key performance indicator of TRBs, influencing power loss and heat generation results increase BT and decrease overall service life. FT mainly results from rolling resistance and sliding friction at roller raceway and roller end rib contacts, while cage friction and lubricant drag generally have smaller contributions. Although numerous analytical, numerical, and experimental studies have investigated the effects of operating and geometrical parameters on FT, limited attention has been given to their combined effects, particularly axial load, rotational speed, and rib convexity. Furthermore, existing FT prediction models generally do not explicitly consider rib convexity, and most optimization approaches focus primarily on minimizing FT without simultaneously maintaining bearing temperature within an acceptable range. These gaps highlight the

need for a systematic experimental investigation, predictive modelling, and multi-response optimization of FT and BT considering both operating conditions and rib geometry.

Chapter 3 – Experimental Setup and Methodology

This chapter presented the experimental methodology for investigating FT and BT of TRB 33118 using a customized test rig equipped with a hydraulic loading system, torque sensor, and SCADA based data acquisition system. The test rig was validated for measurement accuracy and temperature behaviour, and the necessary research considerations and assumptions were established to ensure reliable and repeatable experiments.

Chapter 4 – Design of Experiments

In this chapter experimental results were analysed using FFD and RSM BBD. The L27 FFD and BBD models showed good predictive capability, with predicted R^2 values of 97.10% and 98.16% for FFD, and 95.85% and 96.75% for RSM BBD for FT and BT, respectively. FT increased with axial load and speed, while favourable BT behaviour was observed near 3 μm rib convexity. RSM BBD achieved comparable prediction with only 15 runs compared with 27 runs for FFD.

Chapter 5 – Results and Discussion

This chapter compared Full Factorial optimization, Desirability Function Analysis (DFA), RSM-BBD optimization, and RSM-BBD-based Grey Relational Analysis (GRA) for simultaneous optimization of FT and BT. FFD and DFA identified the practical optimum near 3 μm rib convexity, 2000 kgf axial load, and 400 rpm, while RSM-BBD predicted an optimum of 3.2121 μm , 2000 kgf, and 400 rpm. At this condition, the predicted FT and BT were 7.9639 Nm and 54.5797 °C, respectively, with a composite desirability of 1.000. The RSM-BBD-based GRA produced a comparable optimum at approximately 3 μm , 2000 kgf, and 400 rpm, with predicted FT of 8.125 Nm and BT of 54.5 °C, demonstrating consistency among the optimization approaches.

The confirmation experiments further verified the reliability of the developed models. For the RSM based optimum, the deviations between predicted and experimental values were 0.45–1.64% for FT and 2.54–3.99% for BT. For FFD-based confirmation experiments, the maximum deviations were 4.25% for FT and 3.65% for BT. These results confirm that the developed predictive models and optimization methodology can reliably represent the experimental behaviour within the investigated parameter range.

6.3 Limitations:

- The study is limited to a specific TRB size, geometry, material and surface characteristic, with rib convexity, axial load, and rotational speed investigated within the selected experimental ranges.
- The experiments were conducted using a customized experimental test rig designed for the selected bearing size and operating conditions. Therefore, the applicability of the results to significantly different bearing sizes or operating conditions may require further validation.
- The investigation is based on the assumptions and experimental considerations defined in the study, including lubrication, alignment, loading, wear/run effects, and environmental conditions.
- Hence, direct generalization of the results and developed models to other bearing types, sizes, geometries/profiles, materials, or operating ranges should be made with caution and may require additional experimental and/or analytical investigations.

6.4 Scope of Future Work

- Future studies can extend the FT analysis with combine axial and radial loads on TRBs to enhance model applicability in other real-world applications.
- Coupling FT models with bearing dynamic models can help analyze instability, skidding, and vibration behaviour, especially under high-speed and light load conditions.
- Hybrid approaches combining DOE with machine learning techniques (e.g., ANN-Artificial Neural Network, SVR-Support Vector Regression) may enhance prediction capability and reduce experimental effort compared to conventional DOE.

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List of Publications

1. A. B. Khant, J. K. Dadhaniya, "Influence of Operating Parameters and Rib Convexity on Friction Torque of Taper Roller Bearings," International Journal of Mechanical Engineering, vol. 11, no. 11, pp. 34-46, 2024. DOI: <https://doi.org/10.14445/23488360/IJME-V11I11P104>
2. A. B. Khant and J. K. Dadhaniya, "A Review on Friction Torque of Tapered Roller Bearings," International Journal of Engineering Research & Technology., vol. 14, no. 05, 2025, DOI: 10.5281/zenodo.18106655
3. A. B. Khant, J. K. Dadhaniya, and H. G. Chothani, "Parametric Optimization of Friction Torque and Bearing Temperature in Tapered Roller Bearings Using Response Surface Methodology," Journal of Engineering and Technology for Industrial Applications., vol. 11, no. 54, pp. 120–130, 2025, DOI: <https://doi.org/10.5935/jetia.v11i54.1775>

Appendices

Appendix A - SCADA Sample Reports



Galaxy Bearings Limited
 An ISO/TS 16949 Company
 Survey No. 253, National Highway No. 27,
 Village : Shapar, Dist.: Rajkot - 360024, Gujarat, India
 Website : www.galaxybearings.com
 Email : info@galaxybearings.com

OPERATOR NAME	CHETAN	BEARING CAT. NO	33118 & 33213
BEARING TYPE	TAPER ROLLER	TEST TYPE	FRICTION TORQUE
BEARING SERIES NO	33118 & 33213	TEST TIME	6 hr
AXIAL CYLINDER DIA.	16.0 cm	BEARING 1 MAX TEMP.	190.0 °C
MOTOR SPEED	1010 RPM	BEARING 2 MAX TEMP.	190.0 °C

Sample Report 1

BEARING LIFE TESTING REPORT							
STATION 5							
	Time	ST5_BRG_1 °C	ST5_BRG_2 °C	AXIAL Kg	TORQ N/m	VIBRA	RPM
622	1/13/2024 2:51:39 PM	57.4	51.0	2050.6	17.0	7.3	603.0
623	1/13/2024 2:51:44 PM	57.3	51.4	2050.6	17.0	7.6	603.0
624	1/13/2024 2:51:49 PM	57.9	51.0	2050.6	17.0	7.4	603.0
625	1/13/2024 2:51:54 PM	58.1	51.0	2050.6	16.0	7.4	603.0
626	1/13/2024 2:51:59 PM	58.2	50.9	2050.6	16.0	7.2	603.0
627	1/13/2024 2:52:04 PM	58.2	51.3	2050.6	17.0	7.3	603.0
628	1/13/2024 2:52:09 PM	58.2	51.8	2050.6	14.0	7.9	604.0
629	1/13/2024 2:52:14 PM	58.3	51.5	2050.6	14.0	7.3	604.0
630	1/13/2024 2:52:19 PM	58.6	51.2	2050.6	14.0	7.1	604.0
631	1/13/2024 2:52:24 PM	58.6	52.2	2050.6	14.0	7.3	605.0
632	1/13/2024 2:52:29 PM	58.9	52.0	2050.6	15.0	7.5	603.0
633	1/13/2024 2:52:34 PM	59.2	51.8	2050.6	18.0	8.0	602.0
634	1/13/2024 2:52:39 PM	58.6	51.4	2050.6	17.0	7.4	602.0
635	1/13/2024 2:52:44 PM	58.3	52.1	2050.6	16.0	7.9	604.0
636	1/13/2024 2:52:49 PM	59.5	52.2	2050.6	17.0	7.4	602.0
637	1/13/2024 2:52:54 PM	58.9	52.0	2050.6	17.0	7.2	603.0
638	1/13/2024 2:52:59 PM	58.7	52.7	2050.6	18.0	7.4	601.0
639	1/13/2024 2:53:04 PM	59.7	52.1	2050.6	19.0	7.4	602.0
640	1/13/2024 2:53:09 PM	59.8	52.0	2050.6	19.0	7.3	602.0
641	1/13/2024 2:53:14 PM	60.0	52.1	2050.6	15.0	7.1	603.0
642	1/13/2024 2:53:19 PM	59.9	52.1	2050.6	15.0	7.4	604.0
643	1/13/2024 2:53:24 PM	60.2	52.2	2050.6	16.0	7.4	603.0
644	1/13/2024 2:53:29 PM	61.0	52.7	2050.6	15.0	6.7	604.0

Sample Report 2

BEARING LIFE TESTING REPORT							
STATION 5							
	Time	ST5_BRG_1 °C	ST5_BRG_2 °C	AXIAL Kg	TORQ N/m	VIBRA	RPM
852	1/13/2024 3:10:49 PM	78.3	64.7	2050.6	9.0	6.8	606.0
853	1/13/2024 3:10:54 PM	78.5	64.3	2050.6	9.0	6.9	606.0
854	1/13/2024 3:10:59 PM	78.0	64.8	2050.6	9.0	6.6	607.0
855	1/13/2024 3:11:04 PM	77.6	64.6	2050.6	10.0	6.7	607.0
856	1/13/2024 3:11:09 PM	77.4	64.9	2050.6	10.0	7.2	606.0
857	1/13/2024 3:11:14 PM	77.9	64.6	2050.6	11.0	7.1	605.0
858	1/13/2024 3:11:19 PM	78.1	65.3	2050.6	12.0	6.9	607.0
859	1/13/2024 3:11:24 PM	78.0	64.6	2050.6	13.0	7.3	606.0
860	1/13/2024 3:11:29 PM	77.6	64.5	2050.6	13.0	6.6	605.0
861	1/13/2024 3:11:34 PM	77.6	65.3	2050.6	13.0	6.7	605.0
862	1/13/2024 3:11:39 PM	77.4	65.5	2050.6	13.0	7.1	605.0
863	1/13/2024 3:11:44 PM	78.3	65.2	2050.6	14.0	6.7	604.0
864	1/13/2024 3:11:49 PM	77.8	64.7	2050.6	14.0	6.8	605.0
865	1/13/2024 3:11:54 PM	78.0	65.3	2050.6	15.0	6.9	603.0
866	1/13/2024 3:11:59 PM	78.0	65.0	2050.6	15.0	7.0	604.0
867	1/13/2024 3:12:04 PM	78.5	65.1	2050.6	15.0	7.0	604.0
868	1/13/2024 3:12:09 PM	78.7	65.0	2050.6	15.0	6.8	604.0
869	1/13/2024 3:12:14 PM	78.2	65.6	2050.6	15.0	6.9	604.0
870	1/13/2024 3:12:19 PM	78.6	65.2	2050.6	15.0	7.1	602.0
871	1/13/2024 3:12:24 PM	79.6	65.1	2050.6	15.0	6.8	603.0
872	1/13/2024 3:12:29 PM	79.2	65.3	2050.6	15.0	7.1	604.0
873	1/13/2024 3:12:34 PM	79.1	66.1	2050.6	15.0	6.9	604.0
874	1/13/2024 3:12:39 PM	79.4	65.7	2050.6	16.0	6.9	604.0

Sample Report 3

BEARING LIFE TESTING REPORT							
STATION 5							
	Time	ST5_BRG_1 °C	ST5_BRG_2 °C	AXIAL Kg	TORQ N/m	VIBRA	RPM
277	1/15/2024 10:13:44 AM	54.2	45.1	4101.2	18.0	6.4	399.0
278	1/15/2024 10:13:49 AM	54.6	45.7	4101.2	17.0	6.9	399.0
279	1/15/2024 10:13:54 AM	55.9	48.1	4101.2	17.0	6.7	399.0
280	1/15/2024 10:13:59 AM	55.8	44.0	4101.2	17.0	6.5	400.0
281	1/15/2024 10:14:04 AM	55.3	47.2	4101.2	18.0	6.2	400.0
282	1/15/2024 10:14:09 AM	56.5	46.6	4101.2	18.0	6.7	399.0
283	1/15/2024 10:14:14 AM	55.5	46.8	4101.2	17.0	6.1	400.0
284	1/15/2024 10:14:19 AM	55.5	44.0	4101.2	17.0	6.4	400.0
285	1/15/2024 10:14:24 AM	57.3	45.7	4101.2	17.0	6.9	399.0
286	1/15/2024 10:14:29 AM	55.3	48.1	4101.2	17.0	6.8	400.0
287	1/15/2024 10:14:34 AM	57.3	44.5	4101.2	17.0	6.3	400.0
288	1/15/2024 10:14:39 AM	56.7	45.9	4101.2	17.0	6.7	399.0
289	1/15/2024 10:14:44 AM	55.4	48.3	4101.2	17.0	7.0	400.0
290	1/15/2024 10:14:49 AM	56.2	45.2	4101.2	18.0	6.4	399.0
291	1/15/2024 10:14:54 AM	57.8	48.1	4101.2	18.0	6.3	399.0
292	1/15/2024 10:14:59 AM	56.4	47.3	4101.2	18.0	6.4	399.0
293	1/15/2024 10:15:04 AM	57.5	47.5	4101.2	18.0	6.3	400.0
294	1/15/2024 10:15:09 AM	55.6	47.9	4101.2	18.0	7.2	399.0
295	1/15/2024 10:15:14 AM	56.4	45.4	4101.2	18.0	6.5	400.0
296	1/15/2024 10:15:19 AM	56.1	44.7	4101.2	18.0	6.4	399.0
297	1/15/2024 10:15:24 AM	58.2	45.7	4101.2	18.0	6.1	399.0
298	1/15/2024 10:15:29 AM	55.7	47.5	4101.2	17.0	6.6	399.0
299	1/15/2024 10:15:34 AM	58.0	46.2	4101.2	17.0	6.4	399.0

Sample Report 4

BEARING LIFE TESTING REPORT STATION 5							
	Time	ST5_BRG_1 °C	ST5_BRG_2 °C	AXIAL Kg	TORQ N/m	VIBRA	RPM
829	1/15/2024 10:59:44 AM	62.4	51.9	4101.2	12.0	6.6	605.0
830	1/15/2024 10:59:49 AM	62.4	50.5	4101.2	12.0	6.6	605.0
831	1/15/2024 10:59:54 AM	62.4	51.0	4101.2	12.0	6.7	605.0
832	1/15/2024 10:59:59 AM	62.9	52.9	4101.2	12.0	6.9	605.0
833	1/15/2024 11:00:04 AM	62.1	51.2	4101.2	12.0	6.5	605.0
834	1/15/2024 11:00:09 AM	62.4	52.6	4101.2	12.0	6.5	607.0
835	1/15/2024 11:00:14 AM	62.9	52.3	4101.2	12.0	6.9	606.0
836	1/15/2024 11:00:19 AM	62.7	51.0	4101.2	12.0	6.6	606.0
837	1/15/2024 11:00:24 AM	63.1	53.0	4101.2	12.0	6.7	605.0
838	1/15/2024 11:00:29 AM	62.9	50.9	4101.2	12.0	6.6	606.0
839	1/15/2024 11:00:34 AM	62.8	52.2	4101.2	12.0	6.7	605.0
840	1/15/2024 11:00:39 AM	62.7	51.0	4101.2	12.0	6.8	605.0
841	1/15/2024 11:00:44 AM	62.6	52.8	4101.2	12.0	6.8	606.0
842	1/15/2024 11:00:49 AM	62.8	52.7	4101.2	11.0	6.2	607.0
843	1/15/2024 11:00:54 AM	62.9	52.3	4101.2	11.0	7.0	606.0
844	1/15/2024 11:00:59 AM	63.1	53.0	4101.2	11.0	6.7	605.0
845	1/15/2024 11:01:04 AM	62.5	52.3	4101.2	11.0	7.3	605.0
846	1/15/2024 11:01:09 AM	62.9	52.1	4101.2	11.0	6.6	605.0
847	1/15/2024 11:01:14 AM	62.6	52.7	4101.2	11.0	6.9	605.0
848	1/15/2024 11:01:19 AM	62.6	53.6	4101.2	11.0	6.9	605.0
849	1/15/2024 11:01:24 AM	62.8	53.5	4101.2	11.0	6.4	605.0
850	1/15/2024 11:01:29 AM	62.9	52.0	4101.2	11.0	6.5	607.0
851	1/15/2024 11:01:34 AM	63.7	53.9	4101.2	11.0	6.6	605.0

Sample Report 5

BEARING LIFE TESTING REPORT STATION 5							
	Time	ST5_BRG_1 °C	ST5_BRG_2 °C	AXIAL Kg	TORQ N/m	VIBRA	RPM
1496	1/15/2024 11:55:19 AM	74.2	56.9	4101.2	16.0	6.1	803.0
1497	1/15/2024 11:55:24 AM	74.3	56.7	4101.2	16.0	5.9	804.0
1498	1/15/2024 11:55:29 AM	74.9	56.9	4101.2	14.0	6.2	805.0
1499	1/15/2024 11:55:34 AM	75.0	56.4	4101.2	14.0	5.8	805.0
1500	1/15/2024 11:55:39 AM	74.9	56.3	4101.2	13.0	6.0	804.0
1501	1/15/2024 11:55:44 AM	74.6	56.5	4101.2	13.0	6.0	805.0
1502	1/15/2024 11:55:49 AM	74.7	56.6	4101.2	14.0	6.0	803.0
1503	1/15/2024 11:55:54 AM	74.8	56.8	4101.2	14.0	6.2	804.0
1504	1/15/2024 11:55:59 AM	74.9	56.9	4101.2	14.0	6.2	804.0
1505	1/15/2024 11:56:04 AM	74.7	57.2	4101.2	14.0	6.5	804.0
1506	1/15/2024 11:56:09 AM	75.3	57.6	4101.2	14.0	6.5	803.0
1507	1/15/2024 11:56:14 AM	75.5	56.7	4101.2	14.0	6.6	803.0
1508	1/15/2024 11:56:19 AM	75.2	57.5	4101.2	14.0	6.4	804.0
1509	1/15/2024 11:56:24 AM	75.2	58.2	4101.2	14.0	6.2	803.0
1510	1/15/2024 11:56:29 AM	75.4	57.7	4101.2	13.0	6.4	805.0
1511	1/15/2024 11:56:34 AM	75.2	57.3	4101.2	13.0	6.3	803.0
1512	1/15/2024 11:56:39 AM	75.3	58.1	4101.2	13.0	6.5	805.0
1513	1/15/2024 11:56:44 AM	75.0	58.3	4101.2	13.0	6.3	804.0
1514	1/15/2024 11:56:49 AM	75.4	58.1	4101.2	13.0	6.3	805.0
1515	1/15/2024 11:56:54 AM	75.4	58.1	4101.2	13.0	6.2	805.0
1516	1/15/2024 11:56:59 AM	75.3	58.3	4101.2	13.0	6.6	804.0
1517	1/15/2024 11:57:04 AM	75.2	57.8	4101.2	13.0	6.5	804.0
1518	1/15/2024 11:57:09 AM	75.3	58.2	4101.2	13.0	6.2	804.0

Appendix B - Calibration Certificates

1. Calibration Certificate of Load Cell/ hydraulic pressure gauge (used for axial load measurement)

		Galaxy Bearings Limited AN ISO 9001, ISO 14001 & ISO 45001 CERTIFIED COMPANY Factory / Correspondence Address: Survey no. 253, Meena Cast Gate, National Highway no 27, Village: Shapur, Dist.: Rajkot-360 024, Gujarat, India Phone : + 91 - 2827 - 252401 / 02 Fax : + 91 - 2827 - 252400 Email : info@galaxybearings.com Website: www.galaxybearings.com			
		Galaxy Bearings Limited AN ISO 9001, ISO 14001 & ISO 45001 CERTIFIED COMPANY			
		PRESSURE GAUGE CALIBRATION CERTIFICATE		Format No : F SR 31 Rev.00, Dt. 01/04/2022	
Type of Pressure gauge (UUM) : Hydraulic pressure gauge L.C. : 5 KG/CM2 Range : 250 KG/CM2 Sr No / Identification of Pr gauge : A-38-H-02 M/c No on which the Pr gauge is mounted : A-38				Cali cert No : PG/23/270 Date of calibration : 20/06/2023 Due Dt of calibration : 20/06/2024	
Calibration observation :					
Sr No	Set pressure in master pr gauge	Observation in Pr gauge under calibration	Difference / Error	% Diff / error	Remarks
1	0.0	0.0	0	0.00%	
2	49.6	50.0	0.4	0.16%	
3	99.6	100.0	0.4	0.16%	
4	149.6	150.0	0.4	0.16%	
5	199.6	200.0	0.4	0.16%	
6	249.6	250.0	0.4	0.16%	
7			0	0.0%	
8			0	0.0%	
9			0	0.0%	
10			0	0.0%	
Maximum Difference / Error →			0.4	0.2%	
Detail of Mother instrument used for calibration :					
1. Name of Instrument : Digital master pressure gauge 2. Make of Instrument : Japsin 3. Calibration Cert No & validity : Cert No JI/14/20215/10890 Valid till 16.09.2024 4. Calibration agency : Japsin Instrumentation, Mumbai					
Acceptance criteria :					
1. For Hydraulic pressure gauge : ±8% error 2. For pneumatic pressure gauge : ±5% error					
Calibration result : Acceptable / ~~Not acceptable~~					
Remarks (if any) :					
Calibrated by : 			Approved by : 		
page 01 of 01					
Regd. Off: A-53-54, Parkside Complex, C.G. Road, Emswari, Ahmedabad-380 006, Gujarat, India. Phone No.: +91 - 79 - 29606020, Email : galaxybearings90@gmail.com, Website : www.galaxybearings.com CIN No. L29120 GJ1990 PLC 014385					

2. Calibration certificate of the Tachometer/Rotational Speed Sensor


GALAXY

Galaxy Bearings Limited
AN ISO 18848, ISO 14001 & 45001 CERTIFIED COMPANY
Factory / Correspondence Address:
Survey no. 253, Meena East Gate, National Highway no.27,
Village: Shapur, Dist.: Rajkot-360 024, Gujarat, India
Phone : + 91 - 2827 - 252401 / 02
Fax : + 91 - 2827 - 252400
Email : info@galaxybearings.com
Website : www.galaxybearings.com


GALAXY

RPM METER (UNIT) CALIBRATION CERTIFICATE

Format No : F SR 33
Rev.00, Dt. 01/05/2022

Type of RPM meter (RPM unit) : Electronic display non contact type
L.C. : 1 RPM
Range : 2000 RPM
Sr No / Identification of RPM meter : A-38-R-01
M/c No on which the DUC is mounted : A-38

Condition of DUC : Satisfactory

Calibration observation :

Sr No	Observed RPM in laser tachometer (Mother instrument)	Observed RPM in DUC (Device under calibration)	Difference / Error	% Diff / error	Remarks
1	50	51	1	0.05%	
2	100	102	2	0.10%	
3	250	251	1	0.05%	
4	500	498	2	0.10%	
5	1000	1003	3	0.15%	
6	2000	2005	5	0.25%	
7			0	0.0%	
8			0	0.0%	
9			0	0.0%	
10			0	0.0%	
Maximum Difference / Error →			5	0.3%	

Cali cert No : PG/23/281
Date of calibration : 15/05/2023
Due Dt of calibration : 15/05/2024
Environmental condition :
Humidity : 50±20% Rh
Temp : 25±5 C

Detail of Mother instrument used for calibration :

- Name of Instrument : Digital Tachometer
- Make of Instrument : Maxtech
- Calibration Cert No & validity : Cert No 230826082 Valid till 30.12.2024
- Calibration agency : Alpha calibration services p ltd

Acceptance criteria :

- For Contact type RPM meter : 1% error Max
- For Non contact type RPM meter : 2% error Max

Calibration result : Acceptable / Not acceptable

Remarks (if any) :

Calibrated by : 

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Approved by : 

Regd. Off: A-53-54, Panseema Complex, C.G. Road, Ellisbridge, Ahmedabad-380 006, Gujarat, India.
Phone No.: +91 - 79 - 29606020, Email : galaxybearings90@gmail.com, Website : www.galaxybearings.com
CIN No. L20120 GJ1990 PLC 014385

3. Calibration certificate of the Temperature Sensor (Thermocouple)






Certificate of Calibration

ULR No. → CC345324000008750F		CRF No.: ACS/234-6/24	F/CC/T/04, Issue No.02, Rev. No. 00	
Certificate No.	Date of Calibration	Suggested Due Date	Certificate Issue Date	No. of Page
241118071	18-11-2024	17-11-2025	18-11-2024	1 of 1
Date of Receipt / Ref. No → 18-11-2024		Discipline → Thermal Calibration Temperature		
Name & Address of Customer		GALAXY BEARINGS LTD. Survey No.253, National Highway No.27, Shapar, Dist. Rajkot-360024, Gujarat, India.		
Details of Device Under Calibration (DUC) :				
Instrument Name	Thermocouple	Make/Model	— / —	
ID No.	GB-TC-12	Serial No.	—	
Range/Size	20 to 150 °C	Accuracy/Criteria	— / —	
Type	J Type	Location	—	
Condition of DUC	Satisfactory	Ref. Standard	DKD R 5-1 & ITS-90	
Environmental Condition	Temperature	25 ± 4 °C	Cali. Procedure	ACS/T/CP/04
	Humidity	50 ± 20 % Rh	Cali. Performed	At Lab (Thermal Lab)

Results of Calibration

Reading Observed by DUC in °C	Standard Reading in °C	Error in °C	Expanded Uncertainty (±) in °C
20.415	19.856	0.559	0.53
40.592	39.749	0.843	0.53
60.326	59.896	0.430	0.84
100.124	99.935	0.189	0.84
150.562	149.844	0.718	0.84

Details of Master Instrument Used For Calibration

Nomenclature	ID No.	Certificate No.	Calibrated By	Valid up to
PRT Sensor with 6% DMM	ACS/TL/11	CC350824000000024F	CC-3508	13-03-2025
6% Digital Multi Meter	ACS/ET/01	TSC/24-25/7755-1	CC-2231	29-07-2025

Remarks:

1. The calibration item was not drawn by laboratory.
2. Calibration point and calibration due date are given based on customer requirement.
3. The certificate pertains only to the items calibrated.
4. The reported results are valid at the time of performance of calibration under specified environmental conditions.
5. The uncertainty are for a confidence probability of not less than 95.45 % with multiplying by coverage factor K=2.
6. Temperature liquid baths & dry block calibrator were used as source for generating temperature as well as controlling temperature at specified set points.
7. Equipment used for calibration are traceable to national / international standards.
8. The calibration certificate shall not be reproduced, except in full, without written consent of laboratory.

*** End of Certificate ***

Calibrated By	Reviewed & Authorised By
Jaydip Unagar	Mahek Makadiya (TM) / Mayur Danidhariya (TM)







Maruti Industrial Area, Street No.1, Plot No. 9, 1st Floor, Nr. Maldhan
Railway Crossing, Gondal Chowkdi, Rajkot, Gujarat - 360022.

alphacalibrationservices@gmail.com +91 98243 38248
www.alphagrouplabs.com +91 80000 20499



4. Calibration certificate of the Torque Sensor



GALAXY

Galaxy Bearings Limited
AN ISO 9001, ISO 14001 & 45001 CERTIFIED COMPANY
Factory / Correspondence Address:
 Survey no. 253, Meena Cast Gate, National Highway no.27,
 Village: Shapur, Dist.: Rajkot-360 024, Gujarat, India
 Phone : +91 - 2827 - 252401 / 02
 Fax : +91 - 2827 - 252400
 Email : info@galaxybearings.com
 Website : www.galaxybearings.com



GALAXY

**DYNAMIC TORQUE SENSOR CALIBRATION
CERTIFICATE**

Format No : F SR 35
Rev.00, Dt. 15/05/2022

Type of Torque sensor : Dynamic
 L.C. : 0.1 Nm
 Range : 50 Nm
 Sr No / Identification of Torque sensor : F 70929112
 M/c No on which the DUC is mounted : A-38
 Condition of DUC : Satisfactory

Cali cert No : TM/23/01
 Date of calibration : 01/06/2023
 Due Dt of calibration : 01/06/2024
 Environmental condition :
 Humidity : 50±20% Rh
 Temp : 25±5 C

Ref standard for calibration :
BS 7882:2017

Calibration observation :

Sr No	Observed Torque in Torque transducer (Mother instrument)	Observed Torque in DUC	Difference / Error	% Diff / error	Remarks
1	0.5	0.6	0.1	0.20%	
2	1.0	1.0	0.0	0.00%	
3	5.0	5.1	0.1	0.20%	
4	10.0	10.2	0.2	0.40%	
5	20.0	20.2	0.2	0.40%	
6	30.0	30.2	0.2	0.40%	
7	40.0	40.3	0.3	0.60%	
8	50.0	50.4	0.4	0.80%	
9			0	0.0%	
10			0	0.0%	
Maximum Difference / Error →			0.4	0.8%	

Detail of Mother instrument used for calibration :

- Name of Instrument : Torque transducer
- Make of Instrument : SRMPL
- Calibration Cert No & validity : Cert No 048 Valid till 05.01.2025
- Calibration agency : NPL Delhi

Acceptance criteria :

- For Contact type RPM meter : 2% error Max

Calibration result : Acceptable / Not acceptable

Remarks (If any) :

Calibrated by : 



Approved by : 

page 01 of 01

Regd. Off.: A-53-54, Pariseema Complex, C.G. Road, Ellisbridge, Ahmedabad-380 006, Gujarat, India.
 Phone No.: +91 - 79 - 29606020, Email : galaxybearings90@gmail.com, Website : www.galaxybearings.com
 CIN No. L29120 GJ1990 PLC 014385

Appendix C - Mean and Standard Deviation (SD) Table

Std. Order	A	B	C	FT (Nm)			Mean FT (Nm)	SD FT	BT (°C)			Mean BT (°C)	SD BT
				Run 1	Run 2	Run 3			Run 1	Run 2	Run 3		
1	1	1	1	11	12	13	12	1.00	63	68	64	65	2.65
2	1	1	2	15	13	15	14	1.15	72	72	72	72	0.00
3	1	1	3	13	17	16	15	2.08	78	77	81	79	2.08
4	1	2	1	12	14	14	13	1.15	72	73	76	74	2.08
5	1	2	2	15	14	17	15	1.53	81	78	82	80	2.08
6	1	2	3	17	16	16	16	0.58	87	85	90	87	2.52
7	1	3	1	13	18	14	15	2.65	79	80	77	79	1.53
8	1	3	2	15	18	17	17	1.53	85	84	89	86	2.65
9	1	3	3	20	20	18	19	1.15	98	97	94	96	2.08
10	2	1	1	6	8	10	8	2.00	53	56	56	55	1.73
11	2	1	2	10	10	11	10	0.58	61	60	63	61	1.53
12	2	1	3	10	13	11	11	1.53	65	63	66	65	1.53
13	2	2	1	7	10	10	9	1.73	62	58	64	61	3.06
14	2	2	2	10	13	13	12	1.73	67	65	69	67	2.00
15	2	2	3	15	15	13	14	1.15	73	75	72	73	1.53
16	2	3	1	10	9	11	10	1.00	62	66	64	64	2.00
17	2	3	2	13	14	12	13	1.00	70	71	75	72	2.65
18	2	3	3	14	16	16	15	1.15	81	76	80	79	2.65
19	3	1	1	8	7	6	7	1.00	69	68	65	67	2.08
20	3	1	2	10	9	11	10	1.00	73	75	72	73	1.53
21	3	1	3	12	12	10	11	1.15	82	80	81	81	1.00
22	3	2	1	6	9	9	8	1.73	72	74	77	74	2.52
23	3	2	2	12	13	9	11	2.08	82	77	79	79	2.52
24	3	2	3	13	11	14	13	1.53	85	87	89	87	2.00
25	3	3	1	9	10	8	9	1.00	78	80	77	78	1.53
26	3	3	2	14	12	11	12	1.53	86	84	86	85	1.15
27	3	3	3	12	13	16	14	2.08	91	93	94	93	1.53